Name________________________________

Counting

MathCounts

Solve each:

1. A math conference is presenting a lecture series with six different lecturers. If Dr. Smith's lecture depends on Dr. Jones' lecture, so that Dr. Smith must be scheduled at some time after Dr. Jones, in how many orders can the six lectures be scheduled?
(2006 National Team #9)

1.________

2. On the refrigerator, MATHCOUNTS is spelled out with 10 magnets, one letter per magnet. Two vowels and three consonants fall off and are put away in a bag. If the Ts are indistinguishable, how many distinct possible collections of letters could be in the bag?
(2006 National Sprint #21)

2.________

3. A house with a square floor plan is to have each of its outer four walls finished with brick, stone, stucco, or wood. If no two adjacent outer walls are to have the same finish, in how many ways can the house's walls be constructed? (One such way is brick on the front and back and stone on the left and right.)
(2006 National Sprint #25)

3.________

4. Three-fourths of the students in Mr. Shearer's class have brown hair and six-sevenths of his students are right-handed. If Mr. Shearer's class has 28 students, what is the smallest possible number of students that could be both right-handed and have brown hair?
(2004 National Sprint #4)

4.________

5. Dates can be written as month/day/year using integers 1 through 12 for the month, followed by the day of the month, followed by the last two digits of the year. The date 12/3/45 is an increasing sequence of consecutive digits, while 1/22/34, 1/2/35 and 1/2/03 are not. For how many days in one century is the date an increasing sequence of consecutive digits?
(2003 National Team #6)

5.________

Name________________________________

Probability

MathCounts

Solve each:

1. If two distinct members of the set {2, 4, 10, 12, 15, 20, 50} are randomly selected and multiplied, what is the probability that the product is a multiple of 100? Express your answer as a common fraction.
(2005 National Team #4)

1.________

2. A bag contains 3 tan, 2 pink, and 4 violet chips. If the 9 chips are randomly drawn from the bag, one at a time and without replacement, what is the probability that the chips are drawn in such a way that the 3 tan chips are drawn consecutively, the 2 pink chips are drawn consecutively and the 4 violet chips are drawn consecutively, but not necessarily in the tan-pink-violet order? Express your answer as a common fraction.
(2006 National Sprint #29)

2.________

3. A 3x3x3 wooden cube is painted on all six faces and then cut into 27 unit cubes. One unit cube is randomly selected and rolled. What is the probability that exactly one of the five visible faces is painted?
(2004 National Team #6)

3.________

4. A fair, six-sided die is tossed eight times. The sequence of eight results is recorded to form an eight-digit number. What is the probability that the number formed is a multiple of eight? Express your answer as a common fraction.
(2003 National Sprint #30)

4.________

5. Chris and Ashley are playing a game. First, Chris removes one number from the set {1, 2, 3, 4, 5}. Then Ashley picks two of the remaining numbers at random without replacement. Ashley wins if the sum of the two numbers she picks is *not* prime. Otherwise Chris wins. What number should Chris remove to maximize his chance of winning?
(2003 National Team #3)

5.________

Name______________________________

Statistics: Mean, Median, Mode

MathCounts

Solve each:

1. The math department at a small high school has six classes with 25 students each, four classes with 20 students each, and two classes with 35 students each. No student takes more than one math class. If each student correctly fills out a questionnaire asking for the total number of students in his/her math class, what is the average of all the numbers turned in on the questionairres?
(2006 National Team #7)

1.________

2. A collection of nickels, dimes, and pennies has an average value of $0.07 per coin. If a nickel were replaced by 5 pennies, the average would drop to $0.06 per coin. What is the number of dimes in the collection?
(2005 National Team #5)

2.________

3. Reid took seven tests. On the first five tests he took, he averaged 86 points. On the last three tests, he averaged 95 points. If he averaged 88 points on all seven tests, how many points did he average on the last two tests?
(2004 National Sprint #12)

3.________

4. The arithmetic mean of three numbers x, y, and z is 24. The arithmetic mean of x, 2y, and z-7 is 34. What is the arithmetic mean of x and z? Express your answer as a decimal to the nearest tenth.
(2003 National Sprint #6)

4.________

5. Brad bicycles from home at an average speed of 9 miles per hour until he gets a flat tire. With no way to fix the tire, brad walks his bike back home by the same route, averaging 3 miles per hour. If the entire round trip of biking and walking took a total of 6 hours, what was Brad's average speed in miles per hour for the entire round trip? Express your answer as a decimal to the nearest tenth.
(2003 National Sprint #23)

5.________

Name________________________________

Patterns

MathCounts

Solve each:

1. Consider the sequence $a_1=1$, $a_2=13$, $a_3=135$, ... of positive integers. The k^{th} term a_k is defined by appending the digits of the k^{th} odd integer to the preceding term a_{k-1}. For example, since the 5th term is 13579, then the 6th term is $a_6=1357911$. Note that a_3 is the first term of the sequence divisible by 9. What is the value of m such that a_m is the 23rd term of the sequence that is divisible by 9. (2005 National Sprint #5)

1.________

2. The sequence 12, 15, 18, 21, 51, 81, ... consists of all positive multiples of 3 that contain at least one digit that is a 1. What is the 50th term of the sequence? (2006 National Target #5)

2.________

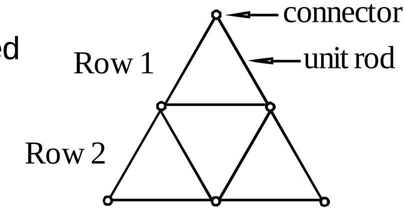
3. What is the sum of the seven smallest distinct positive integer multiples of 9? (2003 National Sprint #4)

3.________

4. The first term of a given sequence is 1, and each successive term is the sum of all the previous terms of the sequence. What is the value of the first term that exceeds 5000?
(2003 National Sprint #13)

4.________

5. A two-row triangle is created with a total of 15 pieces: nine unit rods and 6 connectors, as shown. What is the total number of pieces that would be used to create an eight-row triangle?
(2005 National Sprint #14)



5.________

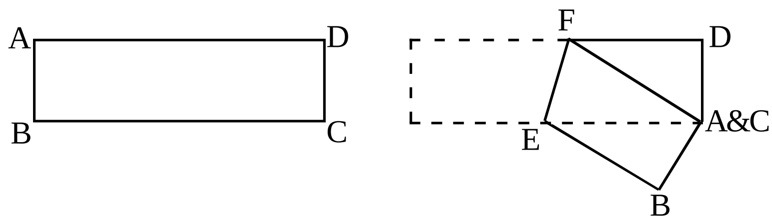
Name________________________________

The Pythagorean Theorem

MathCounts

Solve each:

1. In rectangle ABCD, AB=3 and BC=9. The rectangle is folded so that points A and C coincide, forming the pentagon ABEFD. What is the length of segment EF? Express your answer in simplest radical form.
(2006 National Sprint #23)



1.________

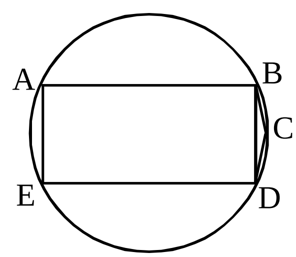
2. For what value a is there a right triangle with sides $a+1$, $6a$, and $6a+1$?
(2003 National Sprint #14)

2.________

3. The square with vertices (-a, -a), (a, -a), (-a, a) and (a, a) is cut by the line $y=\frac{x}{2}$ into congruent quadrilaterals. What is the number of units in the perimeter of each quadrilateral? Express your answer in simplified radical form in terms of a.
(2003 National Sprint #25)

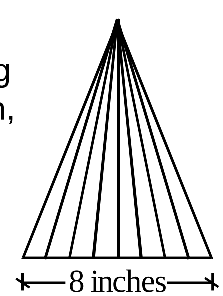
3.________

4. Rectangle ABDE is inscribed in a circle. The lengths of segments AB and AE are 48cm and 20cm respectively. Point C is on the circle, and BC=CD. What is the number of centimeters in the perimeter of pentagon ABCDE? Express your answer in simplified radical form.
(2003 National Sprint #28)



4.________

5. Steve has an isosceles triangle with base 8 inches an height 10 inches. He wants to cut it into eight pieces having equal areas, as shown. To the nearest hundredth of an inch, what is the number of inches in the greatest perimeter among the eight pieces?
(2003 National Target #2)



5.________

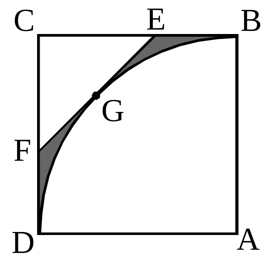
Name________________________________

Area

MathCounts

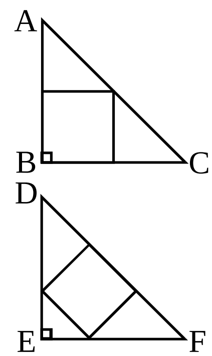
Solve each:

1. Square ABCD has sides of length 1cm. Triangle CFE is an isosceles right triangle tangent to arc BD at G. Arc BD is a quarter-circle with its center at A. What is the total area of the two shaded regions? Express your answer as a decimal to the nearest thousandth.
(2005 National Team #5)



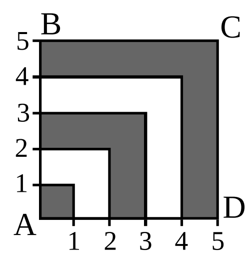
1.________

2. Triangle ABC and triangle DEF are congruent, isosceles right triangles. The square inscribed in triangle ANC has an area of 15 square centimeters. What is the area of the square inscribed in triangle DEF? Express your answer as a common fraction.
(2005 National Target #4)



2.________

3. What percent of square ABCD is shaded? All angles in the diagram are right angles.
(2004 National Sprint #9)



3.________

4. A triangle has vertices at (-3,2), (6, -2) and (3,5). How many square units are in the area of the triangle? Express your answer as a decimal to the nearest tenth.
(2003 National Sprint #11)

4.________

5. Among all triangles with integer side lengths and perimeter 20 units, what is the area of the triangle with the largest area? Express your answer in simplest radical form.
(2006 National Target #4)

5.________

Name________________________________

Three-Dimensional Geometry

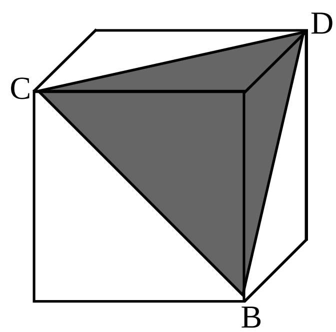
MathCounts

Solve each:

1. A cylinder's height equals the diameter of its base. What fraction of the total surface area of the cylinder is the area of the two bases? Express your answer as a common fraction.
(2003 National Sprint #18)

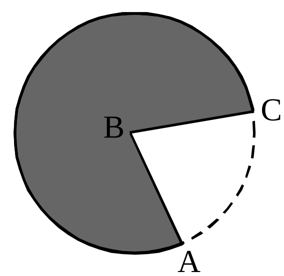
1.________

2. The cube shown has edges of length 2cm. What is the area of triangle BCD? Express your answer in simplest radical form.
(2005 National Sprint #19)



2.________

3. From a circular piece of paper with radius BC, Jeff removes the unshaded sector shown. Using the larger shaded sector, he joins edge BC to edge BA (without overlap) to form a cone of radius 12 centimeters and of volume 432π cubic centimeters. What is the number of degrees in the measure of angle abc of the sector that is not used?
(2004 National Sprint #29)

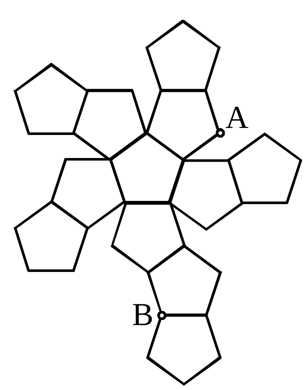


3.________

4. The interior of a right circular cone is 8 inches tall with a 2-inch radius at the opening. The interior of the cone is filled with ice cream, and the cone has a hemisphere of ice cream exactly covering the opening of the cone. What is the volume of the ice cream? Express your answer in terms of Pi.
(2006 National Sprint #19)

4.________

5. An edgy spider walks only along the edges from A to B of the dodecahedron formed by the net shown. What is the number of edges in the shortest path that the spider could take?
(2004 National Target #2)



5.________

Name_______________________________

Proportions, Ratios, and Percents

MathCounts

Solve each:

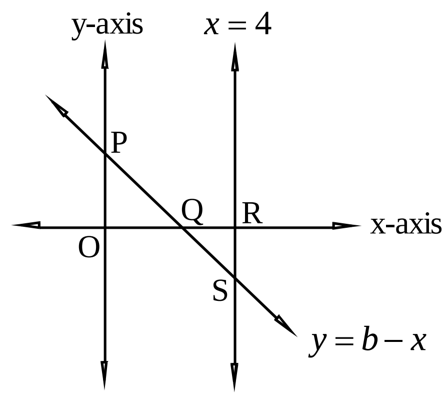
1. If $\frac{x}{y}=\frac{3}{4}$, $\frac{y}{z}=\frac{2}{3}$, and $\frac{z}{w}=\frac{5}{8}$, what is the value of $\frac{x+y+w}{w}$?
(2003 National Target #7)

1.________

2. The price of a stock decreased by 20% in 2001 and then decreased by 20% in 2002. What percent increase in 2003 will restore the price to its value at the beginning of 2001? Express your answer to the nearest hundredth.
(2003 National Team #1)

2.________

3. The line $y=b-x$ with $0<b<4$ intersects the y-axis at P and the line $x=4$ at S. If the ratio of the area of triangle QRS to the area of triangle QOP is 9:25, what is the value of b? Express the answer as a decimal to the nearest tenth.
(2005 National Target #5)



3.________

4. The average heart rate of a shrew is 800 beats per minute, as compared to an elephant with a heart rate of 25 beats per minute. If 1 billion heartbeats is a natural life span for each animal, on average, how many more years do elephants live than shrews? Assume each year is 365 days. Express your answer to the nearest whole number. (2004 National Target #1)

4.________

5. Pipe A will fill a tank in 6 hours. Pipe B will fill the same tank in 4 hours. Pipe C will fill the tank in teh same number of hours that it will take pipes A and B working together to flill the tank. What fraction of th tank will be filled if all three pipes work together for one hour?.
(2003 National Sprint #24)

5.________

Name________________________________

Algebraic Equations

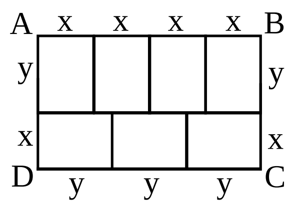
MathCounts

Solve each:

1. Homewood Middle School has 1,200 students, and 730 of these students attend a summer picnic. If two-thirds of the girls in the school and one-half the boys in the school attend the picnic, how many girls attend the picnic?
(2006 National Team #2)

1.________

2. Rectangle ABCD has an area of 336 square inches and consists of seven smaller, congruent rectangular regions. What is the perimeter of rectangle ABCD?
(2006 National Target #2)



2.________

3. If $2x-9y=14$ and $6x=42+y$, what is the value of the product xy?
(2005 National Sprint #11)

3.________

4. The student council sold 661 T-shirts, some at \$10 and some at \$12. When recording the amount of T-shirts they had sold at each of the two prices, they reversed the amounts. They thought they made \$378 more than they really did. How many T-shirts actually were sold at \$10 per shirt?
(2004 National Team #3)

4.________

5. If $a+b=7$ and $a^3+b^3=42$, what is the value of the sum of $\frac{1}{a}+\frac{1}{b}$?
Express your answer as a common fraction.
(2004 National Sprint #25)

5.________

Name________________________________

Number Theory (Factors)

MathCounts

Solve each:

1. Jan is thinking of a positive integer. Her integer has exactly 16 positive factors, two of which are 12 and 15. What is Jan's number?
(2005 National Sprint #20)

1.________

2. What is the product of the two smallest prime factors of 2^{1024} -1?
(2006 National Sprint #26)

2.________

3. For positive integer n such that n<10,000, the number n+2005 has exactly 21 positive factors. What is the sum of all the possible values of n?
(2005 National Target #6)

3.________

4. How many ordered pairs of positive integers (x,y) will satisfy $y = \frac{18}{x}$?
(2003 National Target #1)

4.________

5. N^2 is a divisor of 8!. What is the greatest possible integer value of N?
(2003 National Team #8)

5.________