Proportional Reasoning

Math 8

Solve:

1. $\frac{x}{15} = \frac{2}{5}$

2. $\frac{x}{3} = \frac{2}{11}$

3. $\frac{x}{4} = \frac{7}{15}$

Find more than one way to solve the following problem:
Four men can lift 360 pounds. How many pounds can six men lift?

Method 1: Figure out how much one man can lift and then multiply that by 6.

Method 2: Write and solve a proportion.

Try the following. You should be able to do most of these in your head:

1. It takes three cups of flour to make a cake that serves 6 people. How many cups of flour will it take to make a cake that serves 8?

2. Five centimeters is approximately equal to two inches. What is the approximate length in centimeters of a six-inch ruler?

3. You have a photograph that you want enlarged. The original is 4 inches wide and 6 inches tall. You want it enlarged so that it is 6 inches wide. How tall will the enlarged photograph be?

Ratios are called proportional if they are equal. While the problems above could generally be solved with some basic proportional reasoning, sometimes the numbers are not as easy to work with, and setting up a proportion is more important.

Example:
It takes three and a half cups of flour to make a cake that serves 6 people. How many cups of flour will it take to make a cake that serves 8?

Example:
If 4 gallons will fill 15 one-liter bottles completely, how many 1-liter bottles will it take to hold 7 gallons of water?

Example:
You are enlarging a photograph that is 5 inches wide and 7 inches tall. If you want it to be 8 inches wide, how tall will the enlarged photograph be?

Proportional Reasoning

Math 8

Ratios:
A ratio is a ralationship between two quantities, usually represented as a fraction or by separating the two quantities by a colon.

Example:
The ratio of boys to girls in this classroom is 2:3 (two boys for every three girls). When a ratio represents a part-to-whole relationship, the ratio is usually expressed as a fraction, for example, three out of every five dentists recommend brushing after every meal can be represented as 3/5 of dentists.

Practice: Solve each.

1. The ratio of apples to oranges at a fruit stand is 3:7. If there are 24 apples, how many oranges are there?

2. If 3 out of every 50 students at Ligon are vegetarian, how many vegetarians are there in a school that has 1,050 students?

Think: The ratio of teachers to toddlers in a preschool is 1 to 6. If there are a total of 56 students and teachers at school, how many of them are teachers?

Practice: Solve each.

1. Three out of five Americans surveyed support increased pay for teachers. If 300 people were polled in the survey, how many support an increase in teacher pay?

2. The Carolina Hurricanes ratio of wins to losses in a season was 6 to 5. If the hurricanes played 82 games with 42 wins, how many losses did they have? How many ties?

Think: The ratio of music to commercials on a particular radio station is advertised as 9 to 1. How many minutes of music would you expect to hear in a typical hour?

Name_____ Period _____

Proportional Reasoning

Math 8

Solve each: You may use a calculator. Work on a separate sheet.

1. A recipe calls for 3 eggs to make 4 servings. You want to increase the recipe to serve 12. How many eggs should you use?

2. The three elephants at a zoo drink about 120 gallons of water a day. If the zoo gets two new elephants, how many gallons will they need to provide for all five elephants to get plenty to drink??

3. Michael mixes 8 cups of water, 3/4 cup of sugar, and 2 cups of fresh squeezed lemon juice to make a jug of lemonade. How much water and sugar should he use if he has 3 cups of fresh lemon juice?

water _____ sugar _____

4. The ratio of men to women at a sold-out basketball game is 5:2. If there are 12,000 men at the game, how many women are at the game?

5. A 6 by 10 photograph is being scaled down so that its short dimension is just 2 inches. What will be the length of the longer dimension of the small photo?

6. It takes Roger 15 minutes to complete 20 practice problems. How many minutes per problem is this? How many seconds?

minutes _____ seconds _____

7. A frequent flier program awards 50 points for every 1,000 miles you fly. In one year, you earn 650 frequent flier points. How many miles did you fly in that year?

8. Mr. Jones and Mr. Gaudet have a doughnut eating contest, but Mr. Jones gives Mr. Gaudet an advantage: for every 5 doughnuts Mr. Gaudet eats, Mr. Jones will have to eat six. If Mr. Jones can eat 42 doughnuts in 12 minutes, how many will Mr. Gaudet need to eat to beat him in the competition?

9. One out of every 20 people in a particular city works in the auto industry. If there are 12,350 people who do not work in the auto industry, what is the town's total population?

Similar Triangles

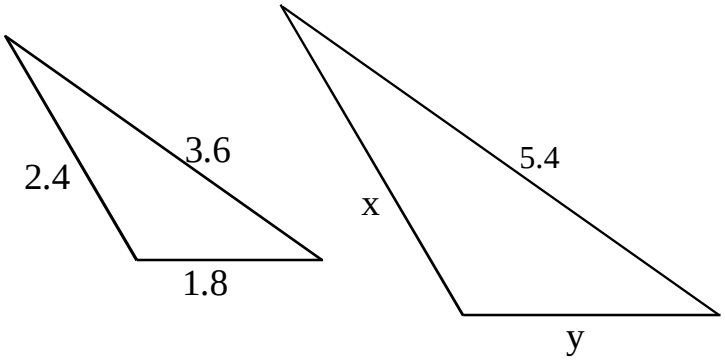
Math 8

Proportions and Similar Triangles:

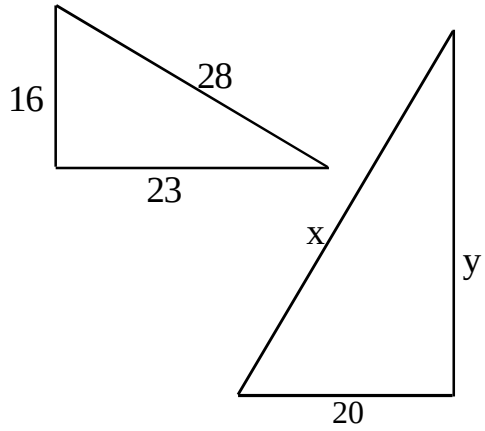
Similar triangles are the SAME SHAPE not the same size.
Corresponding angles are equal.
Corresponding sides are proportional.
You can use proportions to find the length of missing sides.

Ex. Find the length of sides x and y of the similar triangles below:

1.

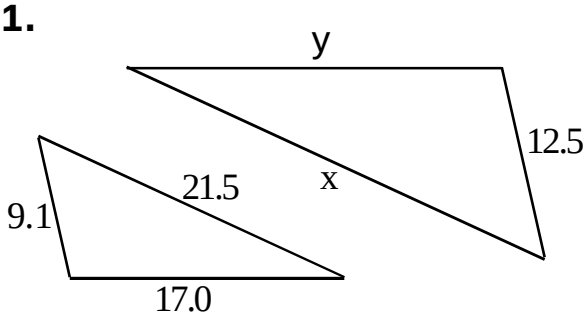


2.

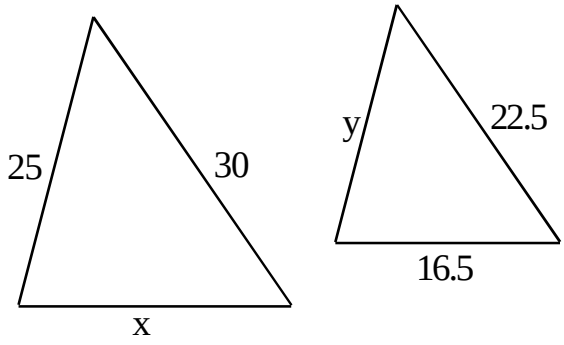


Practice. Find the length of sides x and y of the similar triangles below: Round to the tenth.

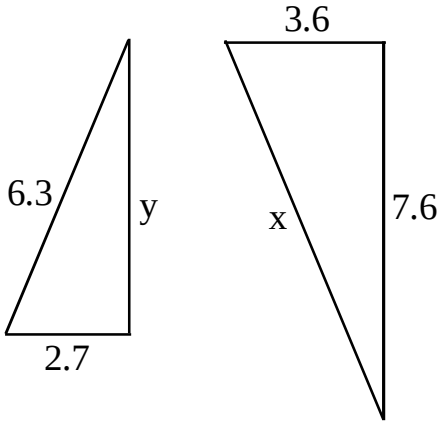
1.



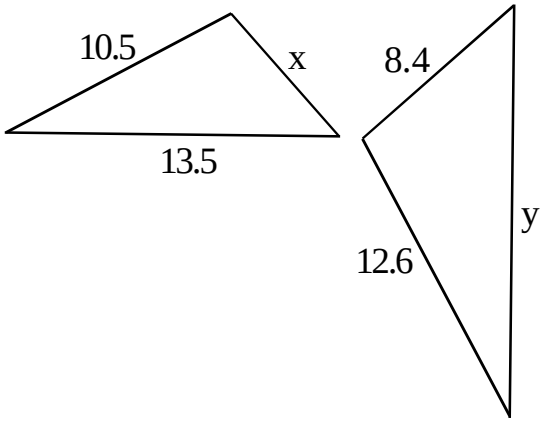
2.



3.



4.



Similar Figures

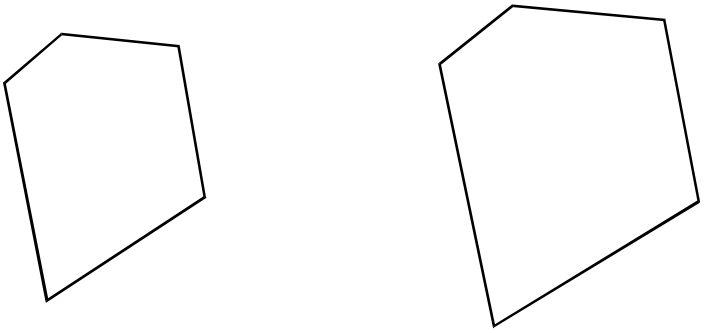
Math 8

Two shapes that are not triangles can be similar.

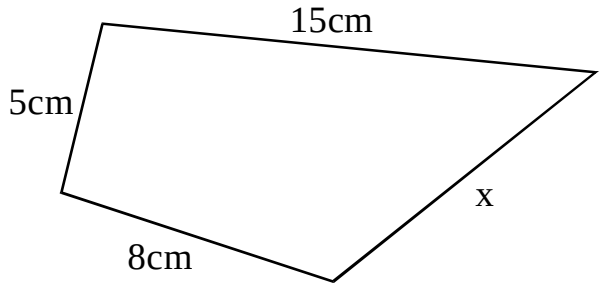
Example: The silhouette of a fighter jet on the left was enlarged to create the image on the right. The wingspan if the smaller jet is 5 inches and the length is 7 inches. If the wingspan of the jet on the right is 11 inches, what is the exact length in inches of the larger plane?

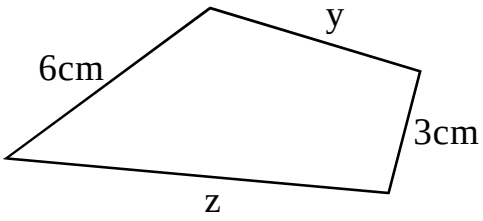
Think of a proportion as a verbal analogy:
Dog is to bark as cat is to _____.
5-inch wingspan is to 7-inch length as 11-inch wingspan is to ___-inch length.

Example: The pentagon on the left has side lengths of 2, 3, 4, 5, and 6cm. The similar pentagon on the right has a short side length of 3.5cm. What are the other side lengths of the larger pentagon?



Example: Find the missing lengths (x, y, and z) of the similar quadrilaterals:





Scale

Math 8

Maps, construction drawings, models, toys, and many other common items use scale.

Most common to most students is the idea of scale on maps. For example, a map of North Carolina might use a scale like 1-inch equals 20 miles. A common problem might ask:

Example: What is the distance in miles between two cities that are 5.25 inches apart on a map of North Carolina if the scale on the map is 1-inch equals 20 miles?

Architectural and engineering scales are different. For example, in architecture 1/4" scale means 1/4-inch equals 1 foot.

Example: The plans for a house are drawn so that 1/2-inch is equal to 1 foot. What is the width in feet of a house that is 17 inches wide in the drawing?

Practice:

1. The scale on a map indicates that 4cm is equal to 60 miles. What is the distance on a map between two cities that are 165 miles apart?

2. On an architectural drawing, the columns on the front of a building are 14 inches tall. The scale of the draing indicates that 3/8" = 1ft. How tall are the columns on the building?

3. A toy motorboat claims to be an exact replic of a real boat, built so that 3/4 of an inch on the toy boat is equal to 1-foot on the real boat. The length of the toy boat is 16.5 inches, what is the actual length of the motorboat?

Practice:

1. Map Scale: 1-inch = 25 miles. How many inches would represent 160 miles?

2. Drawing Scale: 1/4-inch = 1 foot. How many feet would be represented by 11 inches?

3. Model Scale: 3/4 inch = 4 inches. How long would the model of a 12-foot long car be?

Name_____ Period _____

Proportion Review

Math 8

Solve each: You may use a calculator. Show your work.

_____**1.** The scale on a map indicates that 1-inch is equal to 50 miles. What is the real-world distance between two cities that are 360 miles apart on the map?

_____**2.** The shadow cast by a 15-foot phone pole is 9 feet long. What is the height in inches of a dog whose shadow is 3 feet long?

_____**3.** A recipe calls for 1.5 cups of sugar and serves four. You want to make the same recipe for just three people. How many cups of sugar should you use?

_____**4.** A worksheet is 8 inches wide and 11 inches tall, and you want to shrink it so that you can fit two on a sheet of paper. If the new worksheet is just 8 inches tall, what will be its width in inches?

_____**5.** The scale on a set of architectural drawings is 3/8 inch = 1 foot. The windows are 3 inches tall in the drawings. How tall will the wondows be when the building is constructed?

_____**6.** If Randy runs the first three miles of a 26.2-mile marathon in 20 minutes, how long will it take for him to finish at this pace?

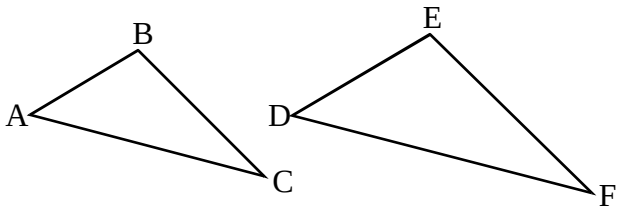
Name_____ Period _____

Proportion Review

Math 8

Solve each: You may use a calculator. Show your work.

_____**7.** Triangles ABC and DEF are similar, with AB = 8cm and DE = 10cm. If AC is 14cm long, what is the length of DF?



_____**8.** The ratio of dogs to cats in the animal shelter is 3:4. If there are 42 animals in the shelter, how many are cats?

_____**9.** A statue of a horse is built so that 16 inches is equal to 1 foot. The tail on the statue is 4 feet long. How long is the tail of the real horse?

_____**10.** In a survey of 140 students, the ratio of students who prefer hamburgers to pizza was 3:4. How many of the 140 students prefer hamburgers?

_____**11.** A model of Monticello was built at a scale of 3/8" = 1 foot. If the actual house is 34 feet high to the top of the dome, what is the height of the model?

Proportions Review

Math 8

Basics:

100. It takes 3 eggs to make a recipe that serves 6. How many eggs will it take to make the same recipe for four people?

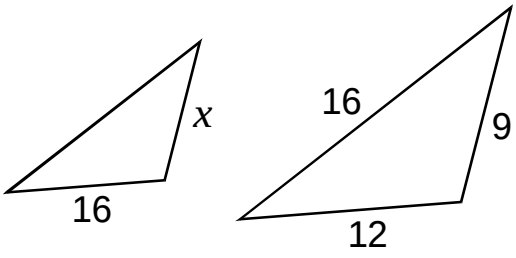
200. Jessica can type 75 words in 3 minutes. How many minutes will it take for her to type a 450-word essay?

300. The ratio of white marbles to red marbles in a bag is 3:5. If there are 30 red marbles in the bag, how many white marbles are there?

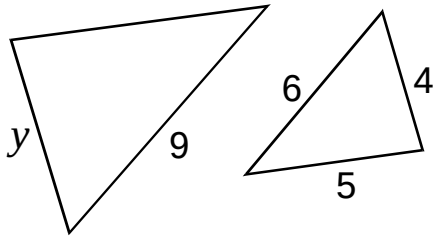
400. There must be 3 chaperones for every 20 students going on a field trip. If 138 people are going on the trip, how many of those going on the trip will be chaperones?

Similarity: Find the missing lengths indicated in each pair of similar figures.

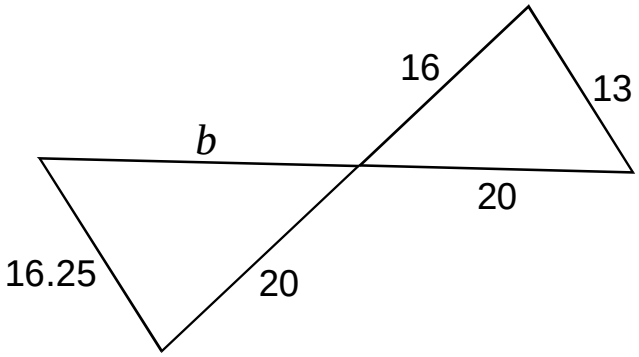
100. Solve for x.



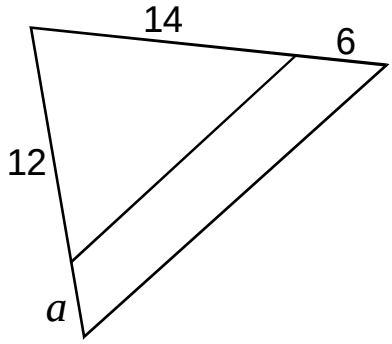
200. Solve for y.



300. Solve for b.



400. Solve for a.



Proportions Review

Math 8

Scale:

100. The scale on a map states that 1-inch is equal to 20 miles. From Raleigh to Durham is 30 miles. How far apart are the two cities on the map?

200. On a scale drawing, 1/4" = 1 foot. What would be the height on the drawing of a 7-foot tall doorway?

300. A model airplane is 14 inches long, built at a scale of 3/4" = 1 foot. What is the length in feet of the actual airplane?

400. A large statue of a warrior is built so that 3 feet of statue represent one inch in real life. If the height of the original warrior was 6 feet, what is the height in feet of the statue?

Mixed Review: Solve each:

100. A six-foot pole casts a 15-foot shadow. What would be the length of Timmy's shoadow if Timmy is 4-feet tall?

200. A photograph is enlarged from 5-inches wide to 8-inches wide. If the original height of the photo was 7.5 inches, what is the new height?

300. The ratio of hardwoods to evergreens in a forest is 3 to 5. If there are 16,000 trees in the forest, how many of the trees are hardwoods?

400. The ratio of men to women eating in a restaurant is 5:6. You look around and see that there are 5 more women in the restaurant than men. How many people are eating altogether?

Name_____ Period _____

Practice Quiz: Proportions

Math 8

Solve each. Simplify all fractions, but you may leave them improper.

1. $\frac{2}{5} = \frac{4}{x}$

1._____

2. $\frac{x-3}{9} = \frac{2}{3}$

2._____

3. $\frac{7}{8} = \frac{x}{3}$

3._____

4. A recipe calls for 5 cups of flour to make 8 servings. How many cups of flour should you use to make enough for just 2 servings?

4._____

5. Melissa earns $20 for every three hours she babysits her little brother. She babysat for 39 hours last month. How much money did she earn?

5._____

6. The scale on a map indicates that 5-inches equals 90 miles. What is the distance on the map between two cities that are 54 miles apart?

6._____

7. The shadow of the school flagpole is 60 feet long. At the same time, you measure the shadow of a stick that is 3-feet long. If the stick casts a shadow that is 5 feet long, what is the height of the flagpole?

7._____

8. The ratio of lions to zebras in a national park is 2 to 35. If there are 250 lions in the park, how many zebra are there?

8._____

9. The construction drawings for a house indicate that 1/2" is equal to 1 foot. If the fireplace is 7 feet wide in real life, how wide will it be in the drawings?

9._____

Name_____ Period _____

Practice Quiz: Proportions

Math 8

Solve each. Simplify all fractions, but you may leave them improper.

10. The instructions on a box of mac-and-cheese suggest using 3 tablespoons of butter and 2 cups of milk. If you increase the recipe, using 4 tablespoons of butter, how much milk should be used?

10._____

11. A bag contains red candies and white candies. The ratio of red candies to white candies in a bag is 3:4. If there are 21 red candies, how many pieces of candy are in the bag altogether?

11._____

12. On a map, you measure the distance from Salem to Florence as 7 inches. You know that Salem is 98 miles from Florence. You then measure the distance betweem Salem and Springfield, which is 9 inches on the map. How many miles is Salem from Springfield?

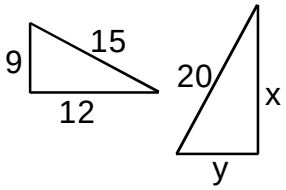
12._____

13. A 5 by 9-inch photograph is enlarged so that the long dimension is 12 inches. What is the short dimension of the enlarged photo?

13._____

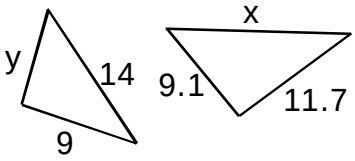
Find the missing length of each:

14.



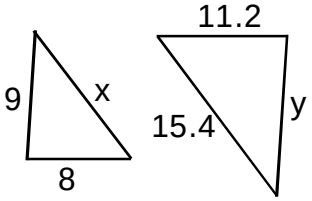
14. x=_____ y=_____

15.



15. x=_____ y=_____

16.



16. x=_____ y=_____

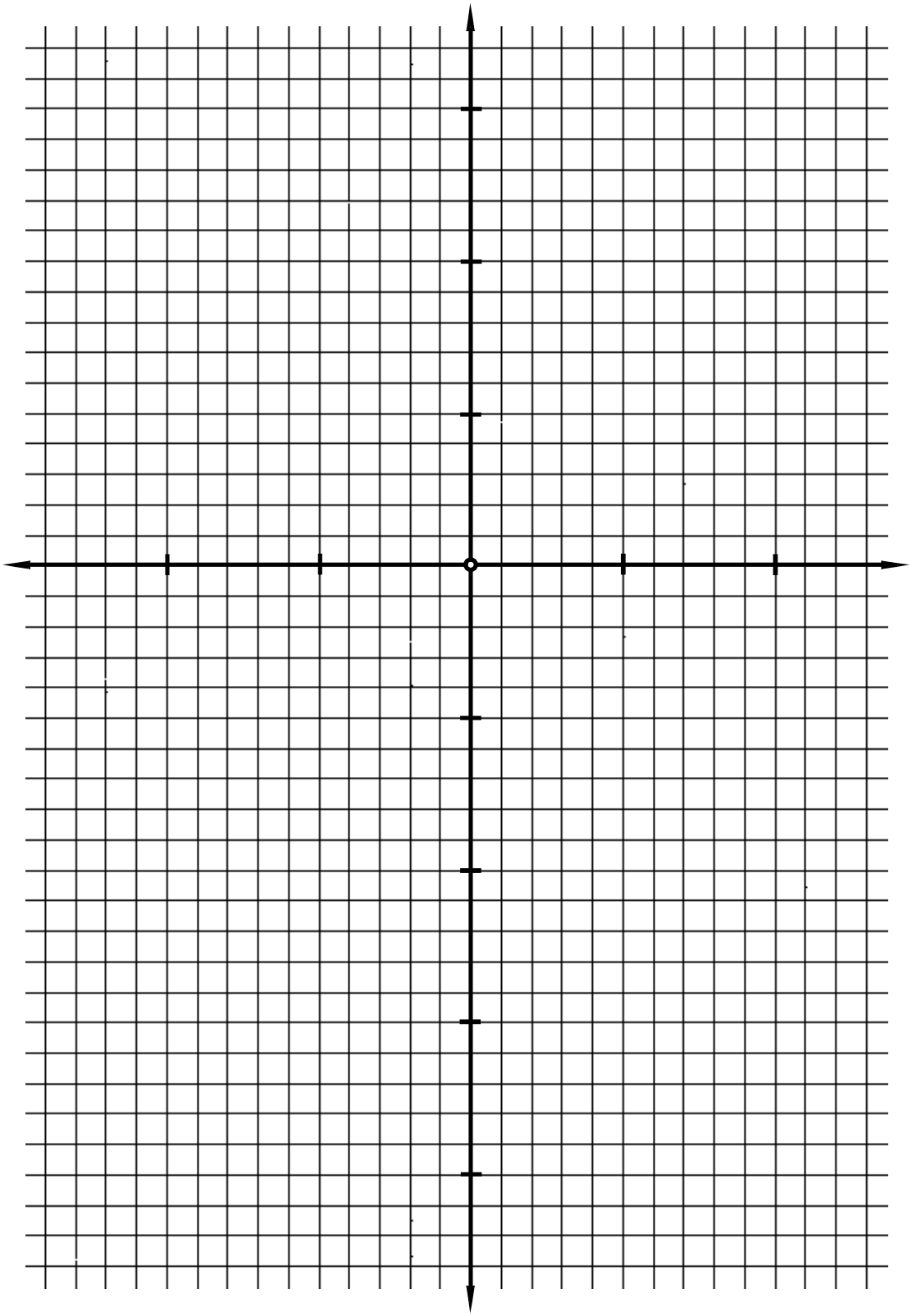
Name_____ Period _____

The Coordinate Plane

Math 8

Graph the coordinates below, connecting as you go.

(-5, 15)
(-8, 11)
(-9, 7)
(-8, 4)
(-8, 2)
(-9, 1)
(-10, 0)
(-10, -1)
(-9, -1)
(-10, -2)
(-10, -3)
(-9, -3)
(-11, -6)
(-10, -7)
(-8, -6)
(-10, -10)
(-7, -11)
(-3, -13)
(0, -16)
(2, -19)
(3, -22)
(5, -22)
(5, -21)
(7, -18)
(8, -16)
(7, -14)
(8, -13)
(6, -11)
(3, -8)
(2, -7)
(2, -6)
(8, -5)
(8, -4)
(7, 1)
(9, 1)
(9, 2)
(7, 7)
(8, 8)
(6, 14)
(6, 15)
(3, 16)
(-1, 16)
(-5, 15)



Can you name this historical figure?

Name_____ Period _____

The Coordinate Plane

Math 8

Graph the points below, connecting as you go.

(-1, 16)
(-3, 16)
(-4, 15)
(-2, 15)
(2, 13)
(2, 10)
(-2, 11)
(-5, 11)
(-7, 12)
(-11, 11)
(-11, 10)
(-7, 10)
(-5, 8)
(-5, 6)
(-8, 5)
(-11, 3)
(-11, 2)
(-7, 4)
(-5, 4)
(-5, -1)
(-9, -2)
(-11, -3)
(-11, -4)
(-9, -3)
(-5, -3)
(-5, -4)
(-9, -5)
(-11, -6)
(-11, -7)
(-9, -6)
(-6, -6)
(-5, -7)
(-7, -9)
(-8, -11)
(-8, -13)
(-7, -14)
(-6, -13)
(-5, -13)
(-5, -15)
(-4, -16)
(-2, -16)
(-4, -15)
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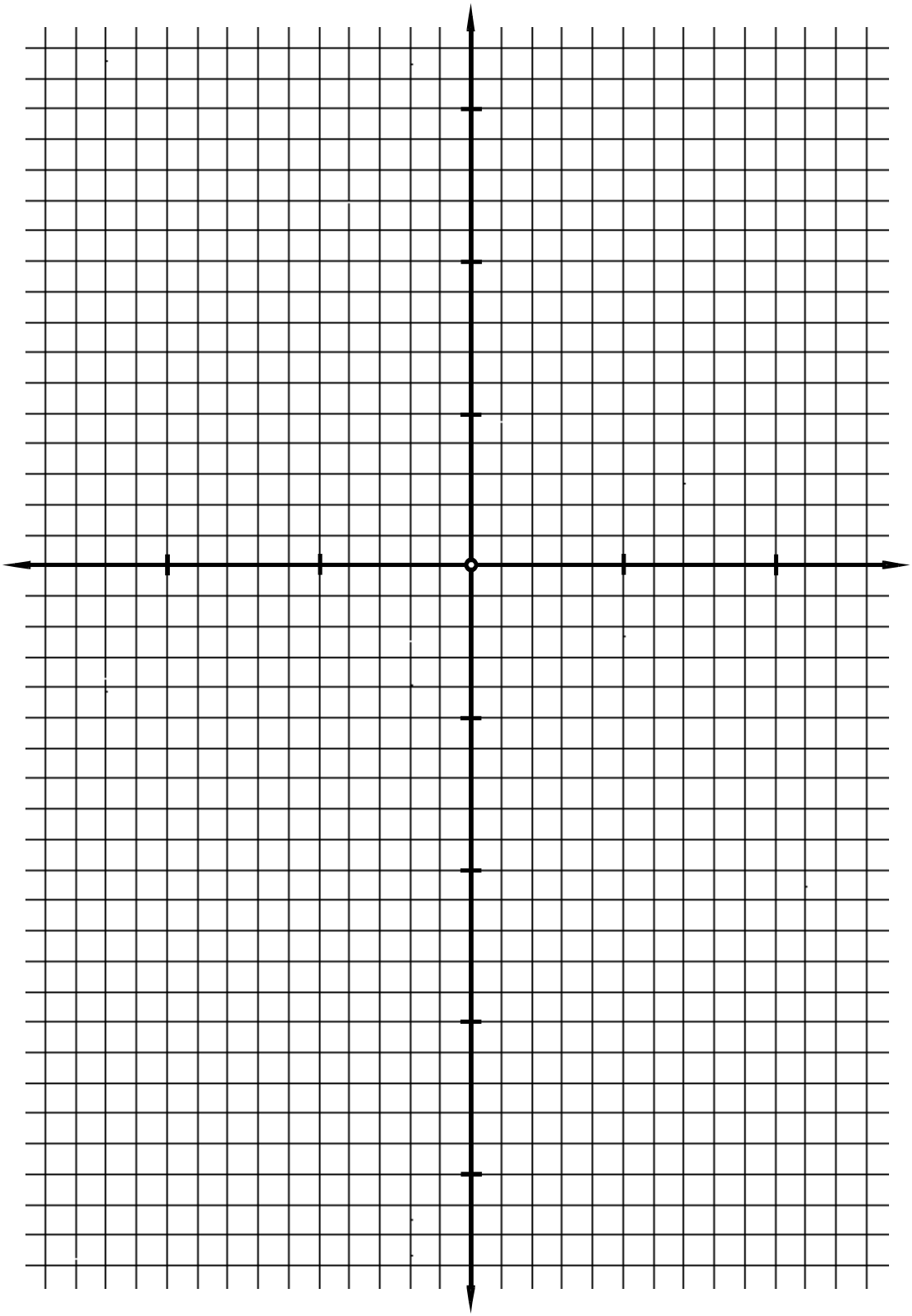
(-3, -11)
(4, -7)
(3, -2)
(3, 13)
(-1, 16)



What animal is this?

Name_____ Period _____

The Coordinate Plane

Math 8

Create your own design on the grid below. Record the coordinates in the blanks. You must use at least 20, but may use as many as you wish.

_____
_____
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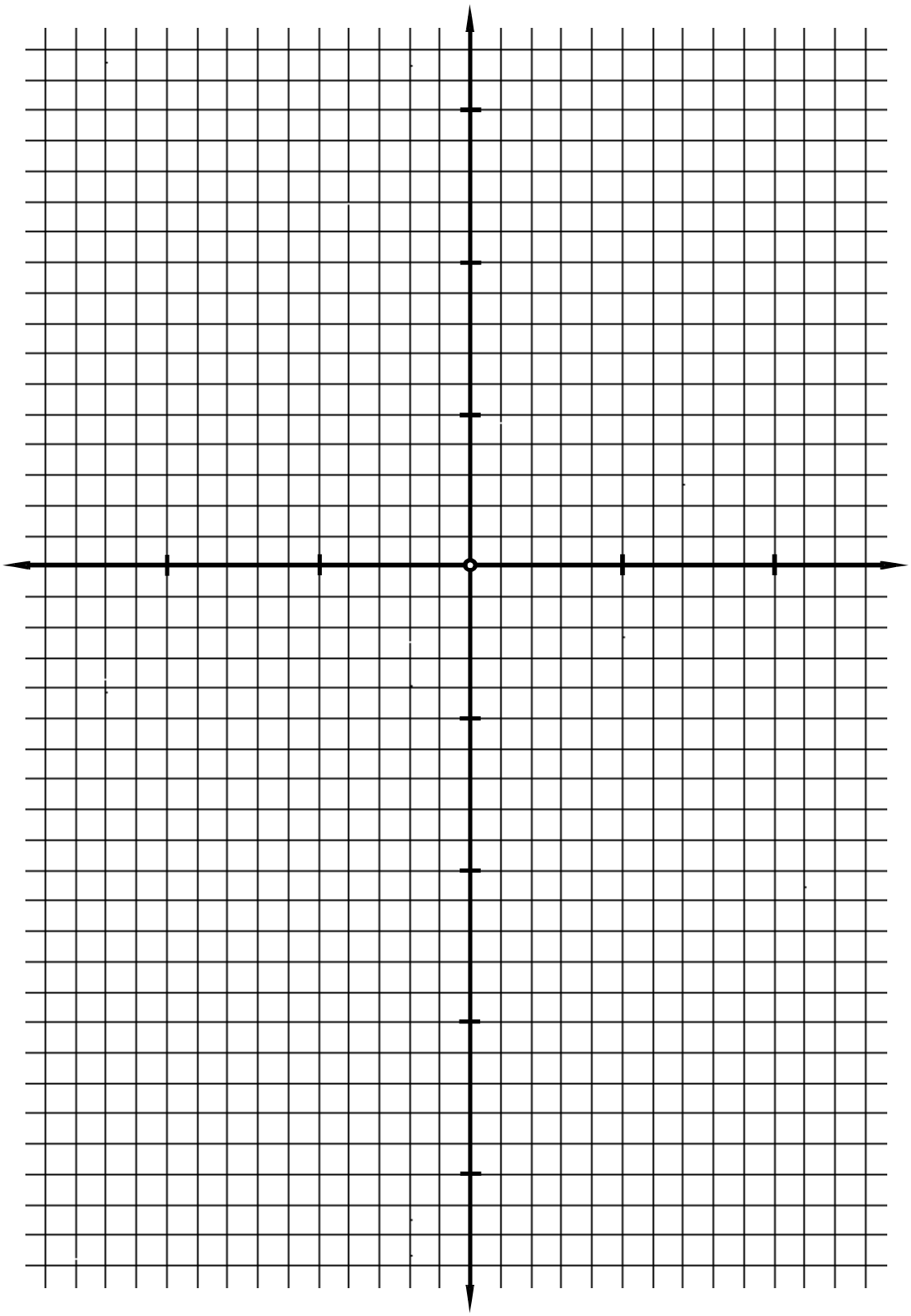


Clue: _____

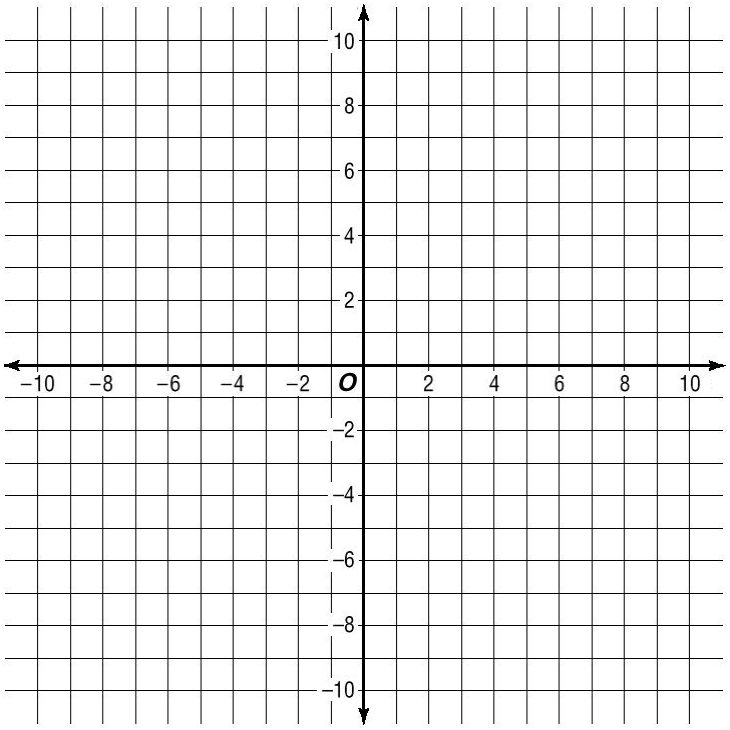
Name_____ Period _____

Review: The Coordinate Plane

Math 8

Review what you know about the coordinate plane below.

Label:
The x-axis.
The y-axis.
The origin.
Quadrants 1 through 4.
Each of the following points:
(1,-4)
(-3,-5)
(2,0)
(0,5)
(9,10)

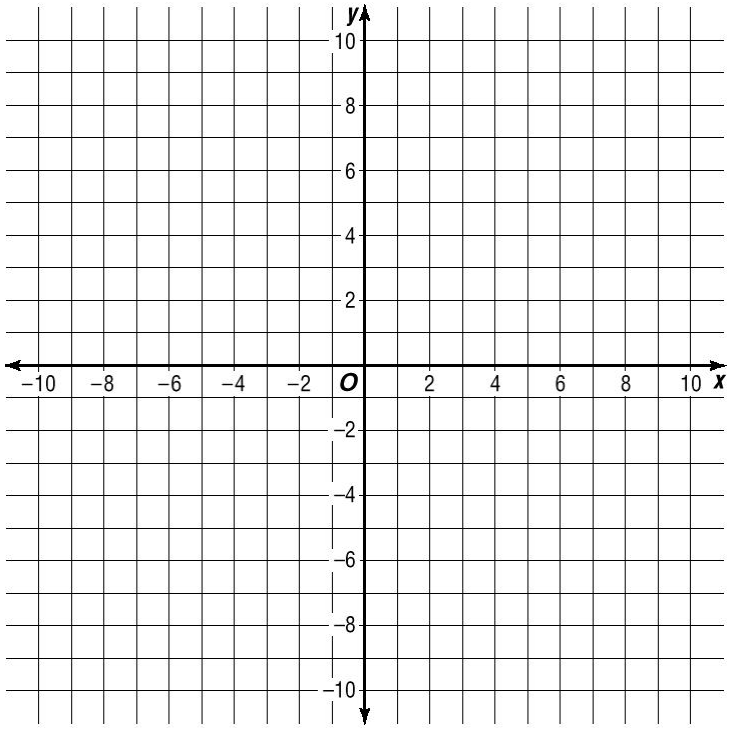


Graphing figures on the coordinate plane is simple.
Graph and connect each set of points below separately.
Connect each set in order and then connect the first and last points in each set.

1. (2,3) (9,3) (9,7) (2,7)
What shape is this?
What quadrant is it in?

2. (-3,-3) (-7,-2) (-8,6) (-4,7)
What shape is this?
What quadrant is it in?

3. (-7,-7) (-3,-9) (6,-6) (2,-4)
What shape is this?
What quadrant is it in?



Dilations on the Plane

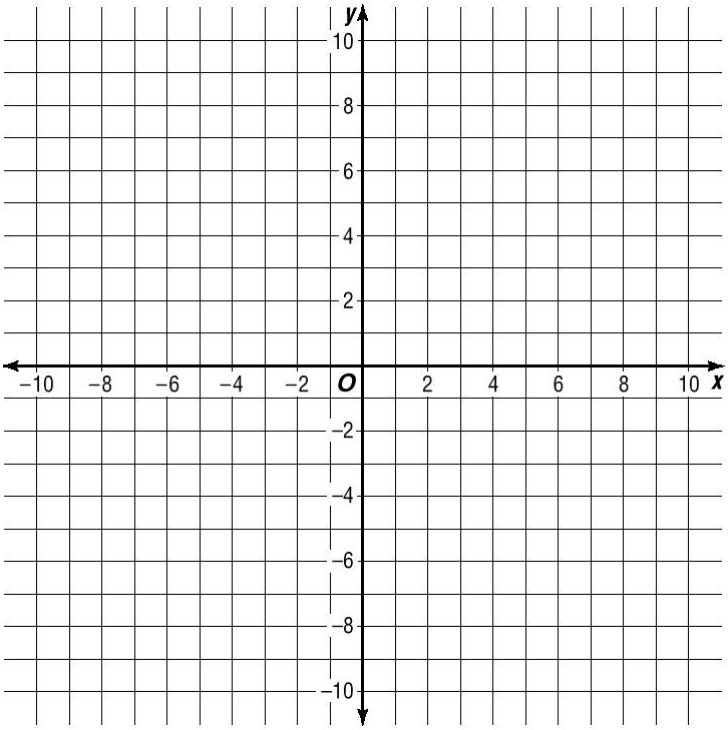
Math 8

A **dilation** is a reduction or enlargement of the original figure. Dilations are similar to the original figure. To create a dilation on the coordinate plane, multiply each coordinate by a **scale factor.**

Practice:
Graph the following triangle.
(-1, 2) (2,1) 3, -2)

Dilate the original triangle with a scale factor of 2.

Dilate the original triangle with a scale factor of 2.5.



Given a figure and its dilation on the plane, it is easy to determine what scale factor was used to create the dilation.

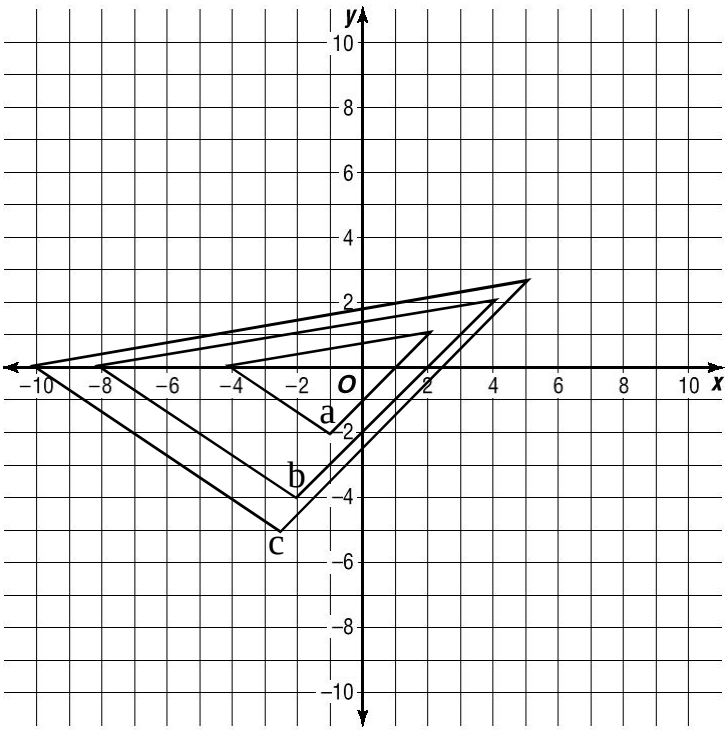
Examples:
What scale factor was used to create each dilation?

1. a to b _____

2. b to a _____

3. a to c _____

4. c to a _____



Name_____ Period _____

Practice: Dilations on the Plane

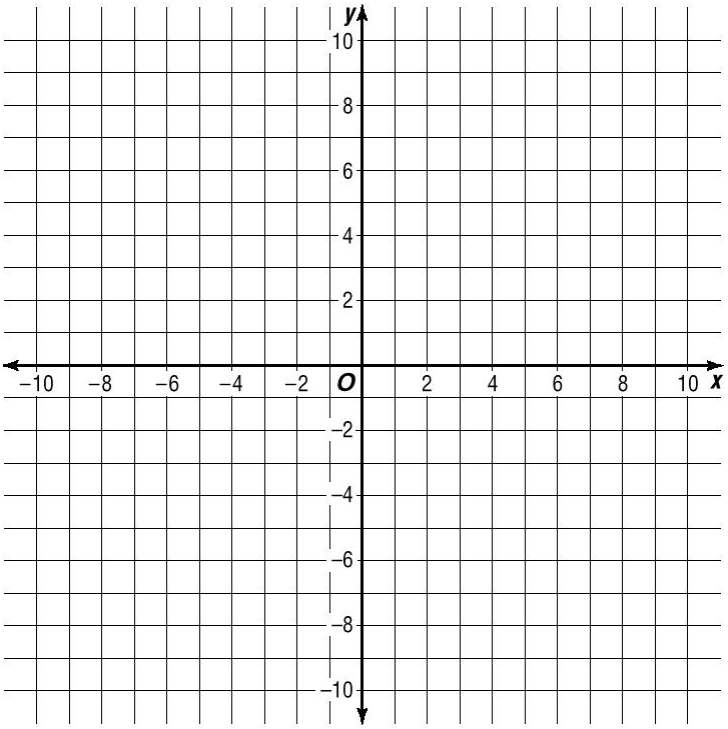
Math 8

Plot each set of points and the dilations listed.

1. (0, -2) (2,2) (4,-2)

2. Graph a dilation with a scale factor of 2.

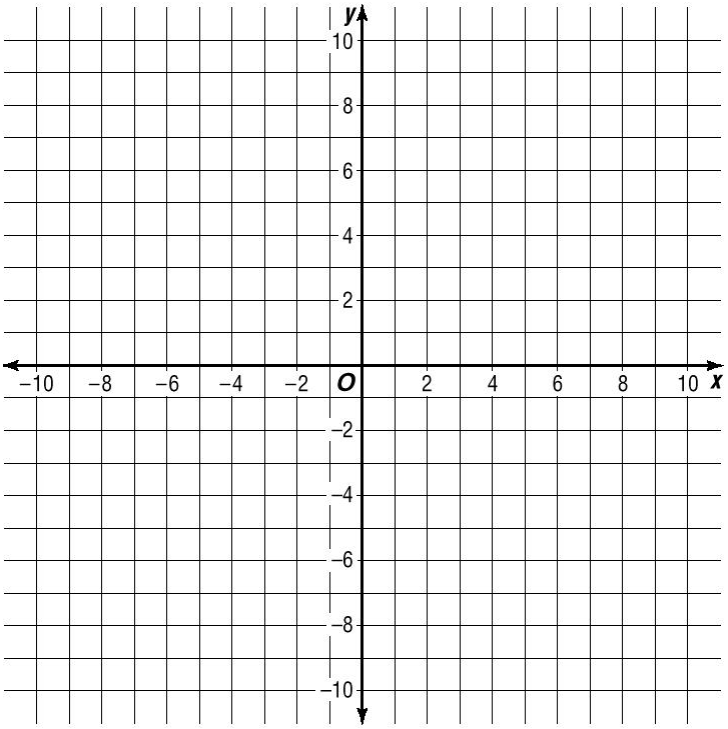
3. Graph a dilation with a scale factor of 2.5



4. (-3, 1) (1,1) (2,-2) (-2,-2)

5. Graph a dilation with a scale factor of 2.

6. Graph a dilation with a scale factor of 3.



Name_____ Period _____

Dilations on the Plane

Math 8

Determine the scale factor used to dilate each pair of figures. Some answers may be fractions.

7. a to b _____

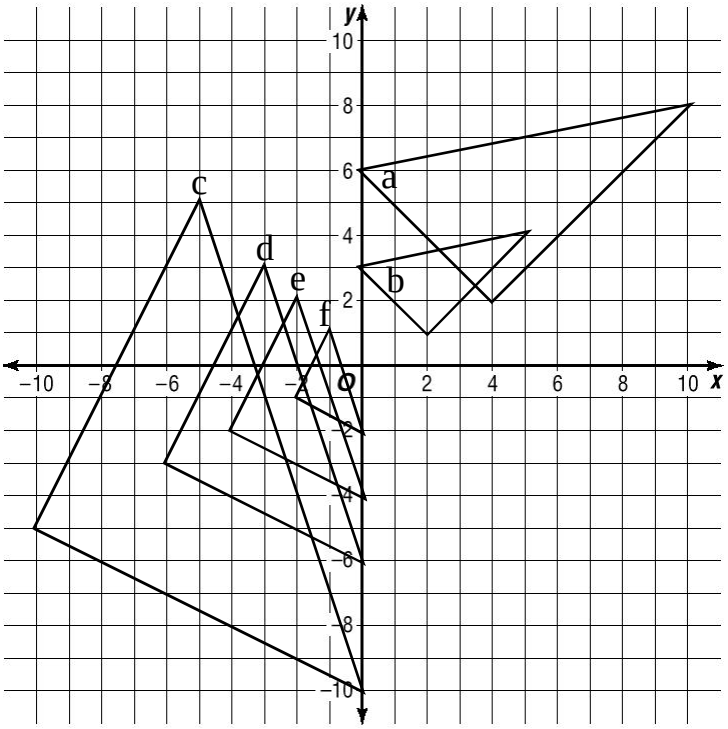
8. b to a _____

9. f to c _____

10. f to d _____

11. e to c _____

12. c to d _____



Complete the following dilations below.

13. Original:
(-4,4) (-8, -8) (8,-4)

14. Dilation with a scale factor of 1/4.

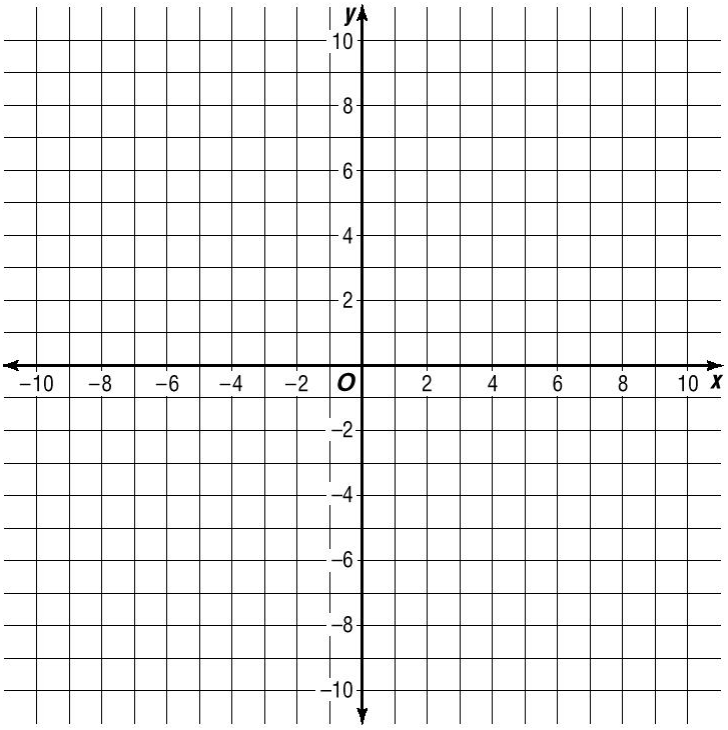
_____

15. Dilation with a scale factor of 0.75.

_____

16. Dilation with a scale factor of 1.2

_____



Conversions

Math 8

It is important that you know how to convert between fractions, decimals, and percents:

Examples: Convert each percent into a fraction and into a decimal.

1. 65% **2.** 2% **3.** 120%

Examples: Convert each decimal into a percent and into a fraction.

1. 0.16 **2.** 0.4 **3.** 1.6

Examples: Convert each fraction into a decimal and into a percent.

1. $\frac{3}{8}$ **2.** $1\frac{3}{5}$ **3.** $\frac{3}{7}$

Percent means per hundred. You can probably come up with a lot of words that use the prefix 'cent' to mean 100. A common method used to convert a ratio (fraction) into a percent is to use a proportion.

Example: Express three-sevenths as a percent. Your percent answer should be a mixed number in simplest form.

Set up the proportion: $\frac{3}{7} = \frac{x}{100}$.

Using cross products, we get $7x = 300$, so $x = \frac{300}{7} = 48\frac{6}{7}$.

Percents can include decimals or as fractions.

For example, how could half of one percent be expressed as:

A percent?

A decimal?

A fraction?

Challenge: Express $7\frac{1}{3}\%$ as a fraction and a decimal.

Name_____ Period _____

Conversions

Math 8

Instructions for all problems:
Use bar notation for repeating decimals. For other long decimals, round to the hundredth.
Do not round terminating decimals like 6.125 or 2.075.
Simplify all fractions. You may use a calculator.

Convert each of the following percents into both a fraction and a decimal.

1. 25%

2. 40%

3. 13%

4. 99%

5. 1,200%

6. 0.75%

7. $2\frac{1}{2}\%$

8. $\frac{1}{5}\%$

9. $100\frac{1}{2}\%$

Convert each of the following decimals into both a percent and a fraction.

10. 0.35

11. 0.1

12. 0.23

13. 2.4

14. 0.009

15. $0.\overline{3}$

Name_____ Period _____

Conversions

Math 8

Instructions for all problems:
Use bar notation for repeating decimals. For other long decimals, round to the hundredth.
Do not round terminating decimals like 6.125 or 2.075.
Simplify all fractions. You may use a calculator.

Convert each of the following fractions into both a decimal and a percent.

Percents that are not whole numbers should be written in fraction form ($2\frac{1}{2}\%$ not 2.5%).

16. $\frac{3}{4}$

17. $\frac{1}{5}$

18. $\frac{3}{8}$

19. $2\frac{9}{10}$

20. $\frac{5}{12}$

21. $\frac{11}{13}$

Proportions and Percents

Math 8

You may have learned the following:

$$\frac{is}{of} = \frac{\%}{100}$$ Is over of equals percent over 100.

This is the useful for very simple problems involving percents:

Examples: Use the Percent Proportion to Solve:

1. What percent is 12 of 40?
2. 6 is 30% of what number?
3. What is 20% of 45?

Practice: Use the Percent Proportion to Solve:

1. What percent of 20 is 12?
2. The number 9 is 25% of what number?
3. What is 15% of 60?

I prefer you use the following variation of **the percent proportion**:

$$\frac{Part}{Whole} = \frac{\%}{100}$$ The part over the whole equals percent over 100.

Examples: Use the Percent Proportion to Solve:

1. There are 60 words altogether on the vocabulary list. Jennifer knows 65% of them. How many of the words does she know?
2. Lewis took 15% of the candy in the bag. If Lewis took 12 pieces, how much candy was there in the bag?
3. 19 of the fish are spotted. If there are 50 fish in the tank, what percent of the fish are not spotted?

Practice: Use the Percent Proportion to Solve:

1. Peter got 80% of the answers on a 40-question multiple choice test correct. How many questions did he answer correctly?
2. Nine of the fifteen teachers are women. What percent of the teachers are men?
3. If 24% of the students in a classroom wear glasses, and there are 19 students who do not wear glasses, how many students are in the classroom?

Name_____ Period _____

Percents and Proportions

Math 8

Solve each using a proportion. Round decimal answers to the tenth.

____**1.** What percent is 7 of 40?

____**2.** What is 15% of 20?

____**3.** 12 is 60% of what number?

____**4.** Ryan made 7 out of 20 free-throw attempts. What percent is this.

____**5.** 90% of the chocolates sold are milk chocolate. If 80 chocolates were sold, how many were milk chocolate?

____**6.** What percent is 15 of 75?

____**7.** There are 12 girls in the class. If 20% of the students in class are girls, how many boys are in the class?

____**8.** 12 is 40% of what number?

____**9.** Six of the numbers on the list are odd numbers. If there are 16 numbers on the list, what percent are odd?

____**10.** The football team has won 35% of its games in the past 10 years. If they have lost 52 games, how many games have they won?

____**11.** What is 42% of 50?

____**12.** 52% of the undergraduate students at UNC are female. If there are 6,292 female undergraduate students at UNC, how many undergraduates attend UNC all together?

____**13.** What percent of 90 is 15? Express your percent answer as a mixed number.

____**14.** Jon read 85% of the book assigned for homework. The book is 320 pages long. How many pages does Jon have left to read?

____**Challenge.** Carrie has won 7 of her first 15 tennis matches. How many wins does she need in a row to improve her winning percent to 75%?

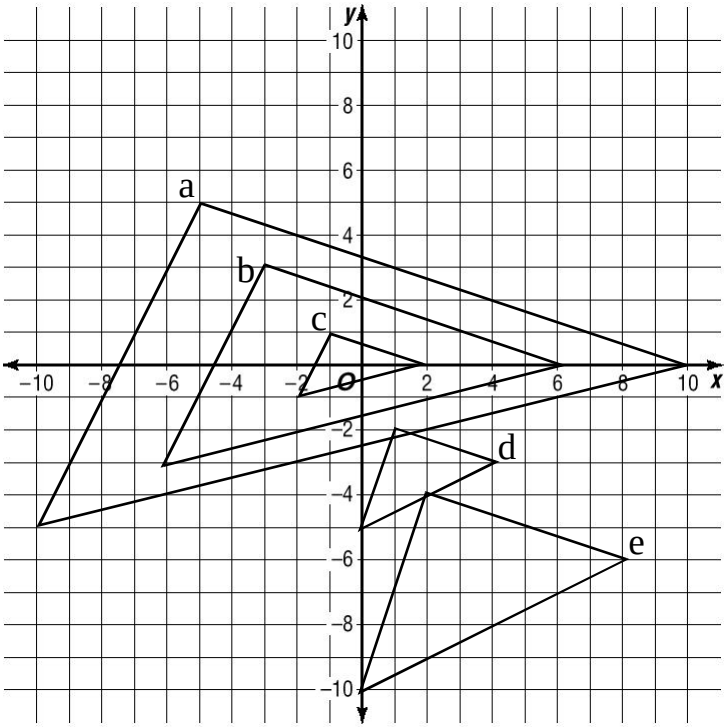
Name_____ Period _____

Half Quiz: Proportions

Math 8

Determine the scale factor used in each dilation below:

12-15.



12. c to a_____

13. b to c_____

14. d to e_____

15. e to d_____

Solve each..

16. What percent of 15 is 12?

16._____

17. What is 2% of 180? Express your answer as a decimal rounded to the tenth.

17._____

18. If 15% of the dogs in the animal shelter are greyhounds and there are 6 greyhounds in the shelter, how many dogs are there at the animal shelter?

18._____dogs

19. A total of 180 teachers attended the state math conference. If 22 of them were Algebra teachers, what percent of the teachers were Algebra teachers? Express your percent answer as a mixed number.

19._____%

20. Gina made 34% of her 3-point shots in this year's basketball season. If she made a record-setting 119 shots from 3-point range, how many 3-point shot attempts did she shoot?

20._____

Percent Change

Math 8

A percent of change is an increase or decrease in value, expressed as a percent. Percent change is one of the most common real-world computations that people perform on a day-to-day basis.

Percent Change:

$$\frac{change}{original} = \frac{\%\ change}{100}$$

Change over original value equals %/100

Examples: Use the Percent Proportion to Solve:

1. The price on a shirt went from $15 to $12. What was the percent discount?
2. Cary has a population of about 100,000 residents, up from only 60,000 just 15 years ago. By what percent has Cary's population increased?
3. Grant grew by 15% in the past two years. If he was 60 inches tall two years ago, how tall is he now?
4. You must pay 9% sales tax on all prepared foods. If a Happy Meal costs $3.25, what is the price after sales tax (rounded to the cent)?

More Percent Change:

$$\frac{new}{original} = \frac{new\,\%}{100}$$

New amount over the original equals new%/100.

This makes some problems much easier when you don't know the change. Try the previous set using this formula.

Examples: Use the Percent Proportion to Solve:

1. Best Buy decreased the cost of its best flat screen monitor by 20%, and it is now being sold for $429.99. What was the cost before the discount (to the cent)?
2. Belle improved her fast pitch speed by 25%. If she was pitching 48mph before, what speed can she pitch now?
3. The population of Bobaloobaville increased by 8% last year. If there are 21,060 people in Bobaloobaville now, how many were there a year ago?
4. Mr. Sharpe improved his mile time by 5%. Last year he could run the mile in 6 minutes. What is his mile time this year?

Percent Change

Math 8

More Percent Change:

Why you need *new over original*:

The most common mistake that people make when solving percent change problems occurs on questions like this:

Ex: Brandon weighs 10% more than Phillip. If Brandon weighs 220 pounds, how much does Phillip weigh?

Many students get 198. Why!?!

Try solving these problems mentally:

1. You have $100. You lose 10% of your money, then gain 10%. How much do you have now?
2. You have $100. You lose 50% of your money. What percent would you need to gain to have $100 again?

This is why we need: $\dfrac{new}{original} = \dfrac{new\,\%}{100}$

Examples: Use the Percent Proportion to Solve:

1. Mr. Batterson invested money in a stock that has increased in value by 44% and it is now worth $5,760. What was the value of the original investment?
2. Deborah improved her quarter mile time by 6.6 seconds, and she now runs the quarter-mile in 59.4 seconds. What percent improvement is this?

Practice: Use the Percent Proportion to Solve:

1. Find the original price on a refrigerator if you paid $588.49 after 7% tax.
2. Sarah improved 100m dash time from 12.21 seconds to 11.48 seconds. By what percent did her time decrease? (to the nearest percent)
3. Antonio is trying to gain weight for football in high school. He weighs 140 pounds now and wants to weigh 161 pounds before tryouts. What percent of weight gain is this?
4. Anna scored a 767 on her most recent math SATs. This is 18% better than her previous score. What was her original SAT score?

Challenge:

Phillip weighs 20% less than Brandon. What percent would Phillip need to gain to weigh the same as Brandon?

Percent Change Shortcuts

Math 8

Decimals and Percents:

Remember, to convert a percent to a decimal, move the decimal point two places to the left.

Easy:

25% = _____ 14% = _____ 6% = _____

Harder:

25,000% = _____ .014% = _____ 6.06% = _____

Try using *is over of* on the following three problems:

1. What percent of 340 is 51?
2. 51 is 15% of what number?
3. What is 15% of 340?

For #3 there is an easier way:

To find a percent of a number, convert the percent to a decimal and multiply.

Ex. What is 12% of 180?

Practice:

1. What is 25% of 190?
2. What is 10% of 34?
3. What is 3.5% of 650?
4. What is 450% of 19?

This is especially helpful in problems involving a percent increase or decrease:

Ex. Find the price after tax on the following items using the given tax. Round to the cent.

1. $40.00 (5%) **2.** $22.50 (7.5%) **3.** $314.99 (3.9%)

Practice:

1. A volleyball is being sold for 25% off. If the original price was $15.96, what is the sale price?
2. The cost of gas increased last week by 2%. If the original cost was $2.50/gallon, what is the new cost?

Challenge: A textbook you need for college has been reduced in price by 30%, and you have a coupon which allows you to save an additional 25%. If tax is 5%, how much will you pay for the book which was originally $40?

Name_____ Period _____

Percents and Proportions

Math 8

Solve. Use the skills we have learned so far to answer the following:

1. What number is 22% of 280?

1. _____

2. What percent of 164 is 123?

2. _____

3. What number is 12% more than 425?

3. _____

4. 20% less than a number is 76. What is the number?

4. _____

5. Tax on a $60.95 pair of shoes is 5%.
How much will you pay for the shoes after tax?

5. _____

6. Corey has 22% more money than Carla. If Corey has $30.50, how much money does Carla have?

6. _____

7. Sears marked up all washer/dryer prices by 8%. If the original price on a Kenmore washer was $350, what is the price after markup?

7. _____

8. Apples are on sale: buy four get one free. This is the same as getting what percent discount?

8. _____

9. After spending $8.40 on lunch Kayla now has 88% of the cash she started the day with in her purse. How much money did she start with?

9. _____

10. Jeremy has 20% more money than Sue, who has 20% less money than Richard. If Jeremy has $57.60, how much does Richard have? (Hint: the correct answer is a whole dollar amount.)

10. _____

Name_____ Period _____

Percents and Proportions

Math 8

Solve. Find the cost of each item after the discount and/or tax. Round answers to the cent.

11. **Skis:** $248.90
Tax: 7%

Price: _____

12. **Envelopes:** $2.95
Tax: 5%

Price: _____

13. **Sofa:** $598.95
Tax: 10%

Price: _____

14. **Cereal:** $4.89
Tax: 5.5%

Price: _____

15. **Computer:** $875.55
Discount: 10%
Tax: 3%

Price: _____

16. **Dining Table:** $185
Discount: 40%
Tax: 8%

Price: _____

17. **Television:** $428.99
Discount: 20%
Tax: 6%

Price: _____

18. **Breakfast:** $14.18
Tax: 4.5%
Tip: 20%

Price: _____

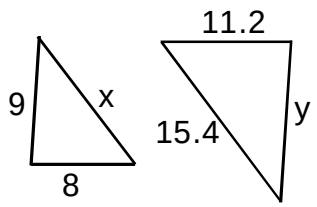
Name_____ Period _____

Practice Quiz: Proportions

Math 8

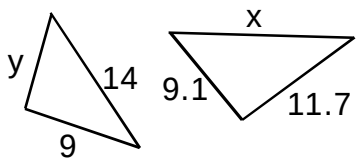
Find the length of the missing sides x and y. Round to the tenth.

1.



1. x_____ y _____

2.



2.x_____ y _____

Solve: Round decimal answers to the tenth.
Use bar notation for repeating decimals. Show all units.

3. What is 15% of 150?

3._____

4. 10 is what percent of 290?

4._____

5. 20 is 125% of what number?

5._____

6. What number is 40% of 12?

6._____

7. What number is 15% more than 80?

7._____

8. 60% less than a number is 33.6. What is the number?

8._____

9. What number decreased by 70% equals 15?

9._____

Name_____ Period _____

Practice Quiz: Proportions

Math 8

Solve: Round decimal answers to the tenth unless noted otherwise. Show all units.

10. A coat is on sale for 30% off. If the original price was $56.95, what is the sale price? Round your answer to the nearest cent.

10._____

11. The tax on a $10.95 pizza is 9%. How much change will you get back if you pay with a $20 bill?

11._____

12. The number of students at Ligon decreased by 6% this year. There are 1,021 students at Ligon this year, how many were here last year (round to the nearest student).

12._____

13. A $65.00 stereo costs $69.55 after tax is added. What percent is the tax amount?

13._____

14. A calculator is on sale for 25% off and now costs $61.74. What was the original price of the calculator to the nearest cent?

14._____

Find the percent of increase or decrease. Round to the tenth of a percent.

15. New price: $24.90
Original Price: $22.91

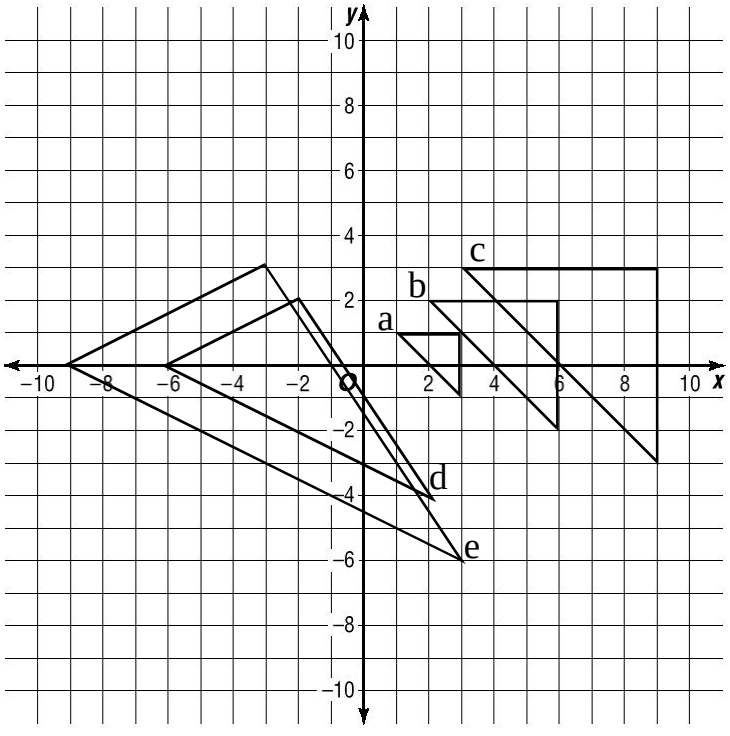
15._____

16. New price: $125.00
Original Price: $320.00

16._____

Determine the scale factor used in each dilation below:

17-20.



17. a to b_____

18. a to c_____

19. d to e_____

20. e to d_____

Percents: Mental Computation

Math 8

Percents:
There are a few percents that are easy to work with that make computation easy: Try these three first.

$10\% = 0.1 = \frac{1}{10}$ $25\% = 0.25 = \frac{1}{4}$ $50\% = 0.5 = \frac{1}{2}$

Asking "What number is 25% of 36?" is the same as "What is 1/4 of 36?" because 25% is 1/4.

Examples: Solve each mentally.
1. What is 10% of 90?
2. 25% of what number is 6?
3. What number is 50% greater than 16?

Practice: Solve each mentally.
1. What is 50% of 44? (20, 22, 66, 88, or 440?)
2. 10% of what number is 20? (2, 22, 100, 200, or 220?)
3. 25% more than what number is 10? (5, 8, 12.5, 25, or 40?)

Practice: Solve each mentally.
1. What is 10% more than 20?
2. What is 50% of 25?
3. 25% of what number is 16?

Some more you should know by heart:

$75\% = 0.75 = \frac{3}{4}$ $20\% = 0.2 = \frac{1}{5}$ $40\% = 0.4 = \frac{2}{5}$

$60\% = 0.6 = \frac{3}{5}$ $33\frac{1}{3}\% = 0.\overline{3} = \frac{1}{3}$ $66\frac{2}{3}\% = 0.\overline{6} = \frac{2}{3}$

Practice: Solve each mentally.

1. What is $33\frac{1}{3}\%$ of 15?
2. What is 20% of 15?
3. What is 60% more than 15?

Percents: Estimation

Math 8

Use the percents we know to estimate.

Examples: Estimate each.

1. Approximate 11% of 39. (2, 3, 4, 5, or 6?)
2. Approximate 24% more than 60. (65, 70, 75, 80, or 85?)
3. A $59 jacket is on sale for 50% off. Approximate the sale price.

Practice: Estimate each.

1. Approximate 35% of 24. (8, 10, 12, 14, or 16?)
2. A cereal costs $3.89, but you have a coupon for 25% off. Approximate the price of the cereal with the coupon. ($2.50, $2.90, $3.20, or $4.20?)
3. You want to leave an 18% tip at dinner for a bill that was $19.81. What whole number of dollars is closest to 18%? ($1, $2, $3, $4, or $5?)

Practice: Match each question with one of the answer choices below.

1. What is 23% of $15?
2. You buy a $20 scarf and use a coupon for 35% off. What is the new cost?
3. You order a $0.99 fries and pay 9% tax. How much do you pay?
4. What is 39% more than $15?
5. 18% of what price is $10?

A. $13.00 B. $55.56 C. $1.08 D. $20.85 E. $3.45

Name_____ Period _____

Practice: Percents and Estimation

Math 8

Solve each: You may NOT use a calculator.

____**1.** What is 10% of 35?

____**2.** What is 25% of 40?

____**3.** What is 50% more than 20?

____**4.** What is 75% of 24?

____**5.** 20 is 25% more than what number?

____**6.** What is $33\frac{1}{3}$% of 21?

____**7.** What is 80% more than 10?

____**8.** What is 10% less than 90?

____**9.** 25% of what number is 11?

____**10.** What is $33\frac{1}{3}$% more than 12?

____**11.** 75% of what number is 18?

____**12.** What is 10% of 200?

____**13.** What is 25% of 200?

____**14.** What is 35% of 200?

Name_____ Period _____

Practice: Percents and Estimation

Math 8

Solve each: You may NOT use a calculator.

_____**15.** The original cost for a pair of shoes was $45, but the cost has been cut by 35%. Which is the best estimate of the new price: $10, $15, $25, $30, or $60?

_____**16.** Tax on a new $59.50 necklace is 9%. What is the best estimate of the price after tax? $60, $65, $70, $75, or $80?

_____**17.** The population of Salem increased by 21% in the past decade. You know that the population ten years ago was 19,709. What is the best estimate of the town's current population? 23,000; 24,000; 25,000; 26,000; or 27,000?

_____**18.** How many questions do you need to get correct to pass (70% or above) a quiz that has 30 questions if each question is worth the same amount?

_____**19.** Tax on groceries is 4.5%. You buy $49.87 worth of groceries. About how much will you pay in tax on the grocery bill? ($1, $1.25, $1.75, $2.25, or $3.75?)

_____**20.** Garrett is 48 inches tall. If he is 79% of his full adult height, approximately how tall will he be when he is fully grown?

Complete Real Numbers Units Here

Math 8

Real numbers unit here, then Blue Diamond Q1 Assessment.

Percents: Percent Equations

Math 8

We have learned to use proportions to solve problems involving percents, for example:

1. If the price on a stereo was increased by 10% and is now $208.89, what was the original price of the stereo?

We have also learned to use decimals in our percent problems, substituting, for example, 25% with 0.25.

1. What number is 25% more than 68?

We can also use decimals to solve problems like the first problem by writing an equation:

1. If the price on a stereo was increased by 10% and is now $208.89, what was the original price of the stereo?

Solution: If we call the original price x, we know that $1.1x = 208.89$, which we can solve for x by dividing both sides by 1.1.

Take a step back: Fill-in the blanks below then solve #5.

1. If a number is increased by 17%, the new value will be _____% of the original value.
2. If I want to increase a value by 17%, I can multiply it by _____.
3. If a number is decreased by 17%, the new value will be _____% of the original value.
4. If I want to decrease a value by 17%, I can multiply it by _____.
5. What is the cost of a $560 diamond ring after 9% tax is added?

Now, try to write an equation to solve each. You may use a proportion to check your work.

1. A number is increased by 17% and is now 93.6. What was the original number.

Percents: Percent Equations

Math 8

Practice: Write and solve an equation for each.

1. 73.6 is 15% greater than what number?
2. The population of Hooville decreased by 24% in the past year. What was last year's population if the current population is 5,001?
3. After 5% tax, the cost of dinner for a family was $57.54. What was the pre-tax cost of the meal?

Practice: Write and solve an equation for each.

1. After a decrease of 8%, the new price of gold is $1,009 per ounce. What was the cost of gold before the drop in price?
2. You use a coupon and save 25% off of your grocery bill. The new bill (with the coupon) is $61.50. How much money did you save?
3. You purchase stock in a company and the stock increases by 36%. The new value of your investment is $6,180. What was your initial investment?

Challenge:

1. You buy a jacket with a 20% off coupon, but the tax on the jacket is 5%. The final bill comes to $49.56. What was the original price of the jacket?

2. The value of an investment has increased by 6% for each of the last three years and it is now worth $4,764. To the nearest dollar, what was the value of the original investment?

Name_____ Period _____

Practice: Percents and Equations

Math 8

Solve each using decimals. Write an equation where necessary,
You may use a calculator.
Round decimals to the hundredth or use bar notation for repeating decimals.

_____**1.** What number is 10% more than 41?

_____**2.** What is 25% of 34?

_____**3.** 36 is 20% greater than what number?

_____**4.** What is 18% less than 20?

_____**5.** What is 31% more than 150?

_____**6.** What is 15% of 418?

_____**7.** 67.5 is 25% less than what number?

_____**8.** What number is 5% less than 70?

_____**9.** What is 55% more than 14?

_____**10.** What is 12% of 49?

_____**11.** 35.2 is 12% less than what number?

_____**12.** What is 5% less than 70?

_____**13.** What is 31% more than 150?

_____**14.** What is 65% of 18?

Name_____ Period _____

Practice: Percents and Equations

Math 8

Solve each using an equation. You may use a calculator.
Round decimals to the hundredth or use bar notation for repeating decimals.
All money answers should be rounded to the cent (always!).

_____**15.** After a 7% was included, a pair of pants cost $29.96. What was the pre-tax price?

_____**16.** The population of Shelbyville increased by 2% in the last year and is now 29,070. How many people lived in Shelbyville last year?

_____**17.** What is the cost of a $395 television after a discount of 15% and tax of 6%?

_____**18.** The value of an investment decreased by 18% and it is now worth $678. What was the value of the initial investment?

_____**19.** With 9% tax and 20% off coupon, the cost of dinner was $51.45. What was the original cost of the meal?

_____**20.** The value of an investment decreased by 50% one year, but the next year it increased by 70%. Overall, by what percent did the value of the stock change? (This one does not require an equation).

Name_____ Period _____

Interest

Math 8

Percents compound, or build on themselves. For example, we have seen that gaining 50% then losing 50% does not get us right back to where we started.

For example, if you invest $1000 at a rate of 10% per year:
In the first year, $100 will be added and the value will grow from $1000 to $1100.
In the 2nd year, $110 will be added and the value will grow from $1100 to $1210.
In the 3rd year, $121 will be added and the value will grow from $1210 to $1331.
Etc.

Notice that you are actually getting more money every year! This is because you are getting a percent of a number that is getting bigger.

Try the following:

1. You invest $2000 in a stock whose value increases by 15% then 14% then 13% in three consecutive years. Which year did you earn the most money on your investment?

Year 1: New value: _____ You gained: _____

Year 2: New value: _____ You gained: _____

Year 3: New value: _____ You gained: _____

2. Here is what many people don't realize, and it can cause a lot of trouble financially. Debt is paid back with interest. Some debt is very expensive. Credit card debt almost always his an interest rate of 18% or more. Say you have $500 in credit card debt at 18% interest and you never pay your bill and you never buy anything else. Use your calculator to find the debt at the end of 1, 2, 3, 4, and 5 years.

Year 0: $500 debt
After 1 Year: _____debt
After 2 Years: _____debt
After 3 Years: _____debt
After 4 Years: _____debt
After 5 Years: _____debt

3. There is an easy way to calculate debt without multiplying by 1.18 over and over again. How could we figure out what the debt would be after 25 years? (hint: Use an exponent.)

After 25 Years: _____debt

Name_____ Period _____

Compound Interest

Math 8

The value of invested money grows or shrinks EXPONENTIALLY.

There is an equation the we can use to model this growth, so that we do not need to continue to add these numbers.

Compound Interest :

$$V = p(1+r)^t$$

V is the Value.
p is the principal or original amount invested.
r is the rate of growth (percent as a decimal - this *can* be negative.)
t is the time - usually in years if we are computing annual growth.

Use the equation above and a calculator to determine what happens to the value of money invested at the given rates and times:

1. $1000 invested at 3% after 10 years. _____

2. $300 invested at 10% after 10 years. _____

3. $500 invested at 7% after 10 years. _____

4. $1000 invested at 3% after 20 years. _____

5. $300 invested at 10% after 20 years. _____

6. $500 invested at 7% after 20 years. _____

Which investment is worth the most after 10 years? 20? Why?

_____

_____

Determine the value of each investment after 15 years:

1. $400 invested with 4% interest. _____

2. $12,450 invested with 11% interest. _____

Name_____ Period _____

Compound Interest

Math 8

For each problem, use:

$$V = p(1+r)^t$$

1. You have $100 invested in an account that earns 7% interest annually, and $200 in an account that earns 4% annually.

a. How much is the $100 investment worth after:

5 years _____ 10 years _____ 20 years _____ 30 years _____

b. How much is the $200 investment worth after:

5 years _____ 10 years _____ 20 years _____ 30 years _____

2. Your grandfather owns two classic cars. His '66 Ford Mustang is valued at $18,000 and appreciates (increases in value) at a rate of about 2% per year. He also owns a '59 Ford Thunderbird worth $12,500 which currently appreciates at a rate of about 6% per year.

a. How much is the Mustang worth after: (round to the nearest dollar.)

5 years _____ 10 years _____ 20 years _____ 30 years _____

b. How much is the Thunderbird worth after:

5 years _____ 10 years _____ 20 years _____ 30 years _____

3. Your other grandfather owns two new cars. His 2007 Honda is worth $16,000 and its value depreciates (decreases) at a rate of 8% per year. His 2007 Ford is worth $21,000 and depreciates at a rate of 11% per year.

a. How much is the Honda worth after: (round to the nearest $100)

5 years _____ 10 years _____ 20 years _____ 30 years _____

b. How much is the Ford worth after:

5 years _____ 10 years _____ 20 years _____ 30 years _____

Name_____ Period _____

Test Review: Proportions/Percents

Math 8

Solving Basic Proportions: Fractional answers should be simplified.

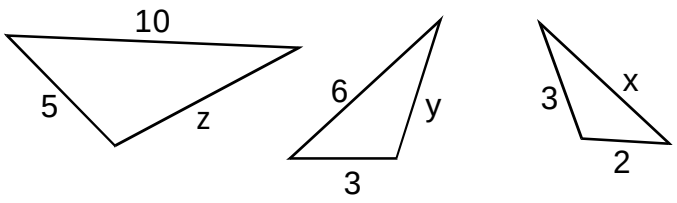
100. $\frac{2}{5}=\frac{10}{x}$ **200.** $\frac{6}{7}=\frac{5}{x}$ **300.** $\frac{3}{x-1}=\frac{2}{x}$

Similar triangles: Find each missing length below in the three similar triangles below.

100: x=_____

200: y=_____

300: z=_____



Proportional Reasoning: Solve each by writing a proportion.

_____**100.** A recipe calls for 5 cups of flour to make 12 servings. How many cups of flour should you use to make enough for just 3 servings?

_____**200.** Paul's shadow is 50 inches long, while Tony's shadow is 60 inches long. If Paul is 5 feet tall, how tall is Tony?

_____**300.** The scale on an architectural drawing indicates that 3/8" is equal to a foot. What would be the length in inches on the drawing of a sidewalk that is 50 feet long? Express your answer as a mixed number.

Percents:

_____**100.** What percent of 10 is 15?

_____**200.** What would be the cost of a $19.95 sweatshirt after 10% discount and 5% sales tax?

_____**300.** On Sunday the price was $57.95. On Monday the price was increased by 30% but then Tuesday the price was decreased by 50%. What was the price after the decrease on Tuesday?

Interest:

_____**100.** If you invest $40 in an account which earns 5% interest each year, how much would your $40 be worth after 2 years?

_____**200.** Every year the value of a new car depreciates (goes down) at a rate of about 10% per year. How much would a $20,000 car be worth in 5 years at this rate? Round to the dollar.

_____**300.** David loans you $4.00 to buy lunch, but you have to pay him back with interest. David offers to let you pay him back at just 1% interest per day (so, if you pay him back tomorrow you would owe him $4 plus 1% interest, or $4.04). If you forget to pay him for a whole year (365 days), how much would you owe David?

Name_____ Period _____

Practice Test: Proportions/Percents

Math 8

Solve each for x: Round decimal answers to the tenth.
Use bar notation for repeating decimals.

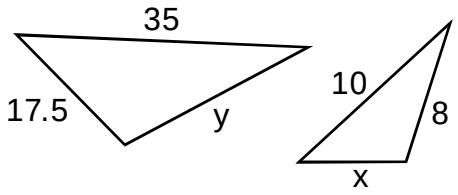
1. $\frac{6}{7} = \frac{21}{x}$

1. x=_____

2. $\frac{6}{x+1} = \frac{2}{3}$

2. x=_____

Solve: Find each missing length in the pair of similar triangles below.



3. x=_____

4. y=_____

Solve each: Round decimal answers to the tenth.
Use bar notation for repeating decimals. Show all units.

5. The scale on a map indicates that 3-inches is equal to 10 miles. What is the distance on the map between two cities that are 25 miles apart?

5._____

6. The shadow cast by a 10-foot pole is 14 feet long. What would be the length of the shadow cast by a 35-foot tall pole?

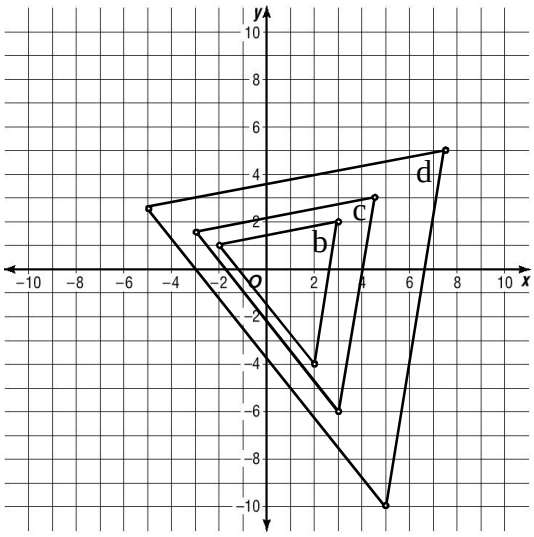
6._____

7. A recipe calls for 3 cups of sugar and will make 20 cookies. You would like to make a batch of 48 cookies. How many cups of flour should you use? Express your answer as a mixed number.

7._____

Determine the scale factor used in each dilation below:

8-10.



8. b to c_____

9. c to d_____

10. d to b_____

Name_____ Period _____

Practice Test: Proportions/Percents

Math 8

Solve each: Round decimal answers to the tenth.
Use bar notation for repeating decimals. Show all units.

11. Express three-fifths as a percent.

11._____

12. What percent of 12 is 5?

12._____

13. What number is 25% more than 56?

13._____

14. There are 12 dogs and 28 cats at a kennel. What percent of the animals at the kennel are dogs?

14._____

15. The number of inches of rainfall in Raleigh is 26% greater than the amount of rainfall received at this time last year. This year's rainfall total is 31.5 inches. How many inches of rain had fallen at this time last year?

15._____

16. What is the cost of a $34 wrist watch after 6% sales tax is included (to the cent)?

16._____

17. You pay for groceries and the final bill comes to $55.60 after you use your Valued Customer Card, which saves you 20% on all purchases. What would have been the cost of the groceries without the card?

17._____

18. You invest $200 in a stock that increases in value by 20%, then decreases in value by 10%. By what percent has the value of your investment increased overall?

18._____

18. The value of your mom's new Audi decreases by 16% every year after she purchases it for $32,000. What is the value of the Audi after 3 years? Round to the nearest dollar.

19._____

20. You have a credit card bill for just $20 that you forget to pay. The annual (yearly) interest rate is 22%, and you don't pay the bill for 20 years. What is the credit card bill with interest after 20 years? Round to the nearest cent.

20._____