Division

## Rewrite as an expression:

1. The sum of seven and $x$.
2. The quotient of $a$ and $b$.
3. Five times the sum of $c$ squared and nine.
4. Nine less than $w$.
5. Twice a increased by nineteen.
6. Two times the sum of a and nineteen.
7. Half the product of $x$ and $y$ decreased by the quantity $x$ plus 4 .

In each case below, replace 'a number' with the variable $\mathbf{n}$ :

1. The sum of seven and a number.
2. The quotient of a number and three.
3. Five times the sum of a number squared and nine.
4. Nine less than a number.
5. Three times a number increased by ten.
6. Two times the sum of a number and four.
7. The product of a number and 3 increased by the number squared.

In the following problems, try to use variables that represent what is being given in the problem (For example, a could be used to represent the number of apples. c could represent the cost. etc.)

1. The number of apples increased by six.
2. Half the cost.
3. Twice the number of cabs increased by three times the number of buses.
4. Nine less than the number of days.
5. Three times as many computers increased by ten.
6. One third of the total number of boys and girls.
7. The cost increased by $20 \%$.

* 

In word problems, the word IS usually means equals.
Terms with no equals sign are called expressions.
If there is an equals sign, it is called an equation.
The following should be written as equations using variables.

1. The sum of seven and a number is 16 .
2. Twenty is three times a number increased by ten.
3. One fifth of the total number of boys and girls is nine more than the number of girls.
4. Four less than the number of pineapples is twice the number of pears.
5. Tom is three years younger than his sister Katie.

The word WHAT usually means USE A VARI ABLE, often we use $x$.
Ex. What is the sum of 2 and a number: $x=2+n$

1. What is the total number of cars and trucks?
2. What is two-fifths of the total cost?
3. What number is three times the sum of itself and seven?

## Defining a variable:

To solve many word problems, you must use a variable to represent an unknown quantity (or quantities). Read the following example:

Margaret has a basket of apples and pears. The number of apples equals three more than twice the number of pears. If there are 15 pieces of fruit altogether, how many apples and pears are there?

Using $p$ for pears and a for apples, write two equations that could help you solve this problem.

## Write three equations:

Amy is five inches taller than James. James is twice as tall as Pamela. Pamela is 41 inches shorter than Amy.

An equation is like a balance. The left side of the equation balances the right because both sides are equal. In order for an equation to remain balanced, what you do to one side you must also do to the other.

Imagine the following scenarios on a playground see-saw:
Group 1
Left side:
1 elephant, 6 lions
Right side:
23 lions

1. How many lions equal the weight of one elephant?

Group 2
Left side:
4 cats, 7 squirrels
Right side:
2. How many squirrels equal the weight of one cat?

## Group 3

Left side:
Right side:
3 geese, 7 ducks 13 ducks, 2 geese
3. How many ducks equal the weight of one goose?

Practice: Try solving the following one-step equations.

1. $-2 x=14$
2. $a-3=-5$
3. $\frac{x}{3}=-9$
4. $\frac{3}{5} x=-12$


## Solving 1-Step Equations:

Three times a number is 21 . What is the number?
Most students have no trouble solving this problem (the number is 7).

Three-fifths of a number is two and a half. What is the number?
This one is much harder to do in our heads, so we must learn to write and solve an equation in a way that works for all problems.

## To Solve a 1-Step Equation:

Undo what is being done to the variable using inverse operations. Multiplication and division 'undo' each other. Addition and subtraction 'undo' each other.

Like the see-saw problems, you must do the same thing to both sides of an equation to maintain balance.

Practice: Try solving the following one-step equations.

1. $x-11=-2$
2. $3+n=-1 \frac{1}{2}$
3. $2 x=-11$
4. $\frac{x}{2}=-\frac{2}{5}$

Practice: Try solving the following one-step equations. Check your work by plugging-in your solution.

1. $x+\frac{3}{4}=-9$
2. $-2+n=9$
3. $\frac{2}{5} x=4$
4. $x \div \frac{2}{3}=-4$

## Solving Multi-Step Equations:

Michael left the house Monday morning with some money in his pocket. He spent $\$ 6$ to take a taxi. He then spent half of what he had left on lunch. Finally, he paid $\$ 1$ to ride the bus home. If he had $\$ 7$ left in his pocket when he got home, how much money did he have to begin the day?
Work backwards in three steps...
How much money did Michael have before he rode the bus?
How much money did Michael have before he paid for lunch?
How much did he have before he paid for the taxi?
To solve multi-step equations, just work backwards.

## To Solve a Multi-Step Equation:

Work in reverse order to undo what is being done to the variable you are solving for.

Example: Solve for $\mathrm{x} . \frac{2 x-11}{3}=-5$.

Consider what happens to the x in this equation:

1. It is multiplied by 2.
2. The result is decreased by 11 .
3. The result is divided by 3.

To undo these steps, work backwards to undo these steps to isolate the variable:
3. Multiply by 3.
2. Add 11.

1. Divide by 2 .

Practice: Solve the following multi-step equations.

1. $3 x-2=7$
2. $\frac{7+4 x}{3}=5$

Practice: Solve the following multi-step equations.

1. $3(x-2)+1=13$
2. $\frac{6-x}{2}=5$

Solve each of the followingfor $x$.
Add, Subtract, Multiply or Divide both sides to isolate the variable.

1. $-2 x=4$
2. $6-2 x=10$
3. $\frac{6-2 x}{3}=30$
4. $\frac{6-2 x}{3}-1=29$
5. $x-2=11$
6. $3(x-2)=33$
7. $3 x=39$
8. $3 x-6=33$
9. $\frac{4-2 x}{3}=6$
10. $5-2(x-6)=11$
11. $\frac{2}{3} x-1=5$
12. $\frac{2}{3}(x-1)=4$

Sometimes an equation will have the variable on both sides:
Example: Solve for $\mathrm{x} . \quad x-5=2 x$.
If possible, you should begin by moving the variable to one side of the equation.

Examples: Solve the following for x .

1. $3 x-2=7 x$
2. $2 x-8=5 x+7$

Sometimes you must do some work before you move the variable:
Examples: Solve the following for x .

1. $\frac{3 x-2}{4}=x$
2. $2(3 x-5)=4 x$

Check: Plug-in your solution to check your work.

Practice: Solve the following for x .

1. $x-10=3 x$
2. $7 x-8=3 x+8$
3. $3(x-2)=6 x$
4. $\frac{3 x+14}{2}=5 x$

Practice: Solve the following.

1. What fraction is equal to five less than four times itself?
2. Paul goes to the store with dollars. He spends nine dollars at the pharmacy and half of his remaining money on groceries. When he returns home he has only one-fifth of the money he started with. How much money did he start with?
3. $12^{2}$
4. $0.6^{2}$
5. $(-9)^{2}$ 4. $\left(\frac{10}{11}\right)^{2}$

Squares and Square Roots are Inverse Operations.
If $x^{2}=y$, then $x$ is a square root of $y$.
Every positive number has two square roots.
You will only need to indicate the positive square root of a number unless this symbol appears before the radical:

$$
\pm \sqrt{ } \text { We will come back to this later in the year. }
$$

Examples: Find each square root.

1. $\sqrt{25}$
2. $\sqrt{144}$
3. $\sqrt{0.25}$
4. $\sqrt{\frac{4}{9}}$

Practice: Try some more decimals and fractions.

1. $\sqrt{0.81}$
2. $\sqrt{1.44}$
3. $\sqrt{\frac{49}{100}}$
4. $\sqrt{\frac{1}{121}}$

Practice: Try estimating each square root to the nearest whole number.

1. $\sqrt{80}$
2. $\sqrt{50}$
3. $\sqrt{15}$
4. $\sqrt{2,499}$
5. The area of a square is $120 \mathrm{~cm}^{2}$. Approximate the length of its sides to the nearest centimeter.
tax
Examples: Try solving some equations that includes a square root.
6. $\sqrt{x}=5$
7. $\sqrt{x}-2=7$
8. $\frac{\sqrt{x}}{5}=2$

To solve an equation which includes a square root:

1. Isolate the radical (square root).
2. Square both sides.
3. Finish solving if necessary.

Examples: Solve for x .

1. $\sqrt{2 x}+3=7$
2. $2 \sqrt{x-3}=14$

Practice: Solve for x .

1. $\sqrt{x}-7=3$
2. $2 \sqrt{x}-5=3$
3. $\sqrt{x-2}=7$
4. $\frac{2 \sqrt{13+x}}{3}=8$

Practice: Word problems.

1. Five more than the square root of a number is 7 . What is the number?
2. Ian selects a number at random. Alex adds 7 to the number. Briana divides this result by 2. Finally, Alison takes the square root of the result. If all of the computations are correctly and Alison's answer is 10, What was Ian's original number?

Practice: Write the first 15 perfect squares.

| $1^{2}=$ | $2^{2}=$ | $3^{2}=$ | $4^{2}=$ |
| :--- | :--- | :--- | :--- |
| $6^{2}=$ | $7^{2}=$ | $8^{2}=$ | $9^{2}=$ |
| $11^{2}=$ | $12^{2}=$ | $13^{2}=$ | $14^{2}=$ |
|  |  | $10^{2}=$ |  |

Practice: Find each square root.

1. $\sqrt{81}$
2. $\sqrt{144}$
3. $\sqrt{121}$
4. $\sqrt{225}$
5. $\sqrt{49}$
6. $\sqrt{400}$

Practice: Find each square root. Please do NOT use a calculator. These can all be done in your head using the square roots you know.
7. $\sqrt{0.25}$
8. $\sqrt{1.69}$
9. $\sqrt{0.64}$
10. $\sqrt{810,000}$
11. $\sqrt{0.01}$
12. $\sqrt{4,000,000}$

Estimate: To the nearest whole number, OR, you may list the two numbers that the square root is between. Again, NO calculators.
13. $\sqrt{45}$
14. $\sqrt{175}$
15. $\sqrt{199}$
16. $\sqrt{905}$
17. $\sqrt{175}$
18. $\sqrt{999,999}$

Practice：Solve each for $x$ ．Plug－in your answer to check that your solution is correct．

19．$\sqrt{x}=13$
20．$\sqrt{x}-7=1$

21． $4+\sqrt{x}=7$
22．$-13-\sqrt{x}=7$（careful，check your work）

23．$\sqrt{3 x}=9$
24．$\sqrt{\frac{x}{3}}-7=-3$

25．$\frac{2+\sqrt{x}}{4}=3$
26．$\sqrt{\frac{x-15}{2}}-3=2$

Challenge：If I increase the square root of Phil＇s age by 10 and then take the square root of the result，I get 4．How old is Phil？

First, make sure you can solve each of these in your head:

1. $25 \cdot 100$
2. $100 \cdot 7.09$
3. $1,000(1.003)$

## Eliminating Decimals:

Equations like the following can be difficult to solve. There is an easy trick to make these easier. What could you do to the equations below to make them easier to work with?

1. $0.9 x-4.3=2$
2. $3.5-1.2 x=2.3$

You can multiply both sides of the equations below by 10.
This moves the decimal points to the right one space.
The resulting equations are easier to solve:

$$
\begin{array}{ll}
\text { 1. } 9 x-43=20 & \text { 2. } 35-12 x=23
\end{array}
$$

What would be a good first step for solving this equation?

$$
0.1 x-0.35=2.1
$$

Multiply the whole thing by 100, then solve it (remember, this moves each decimal two spaces to the right):

$$
100(0.12 x-0.78=2.1) \text { gives us } 12 x-78=210 .
$$

Practice: Solve for x .

$$
\begin{array}{ll}
\text { 1. } 0.4 x-3.4=2.2 & \text { 2. } 4.05=1.1 x+0.2
\end{array}
$$



## Removing fractions:

What could you multiply the whole equation below by to make it easier to solve?

$$
\frac{2}{5} x-\frac{3}{5}=\frac{1}{5}
$$

It is not always so obvious:
Look for the least common denominator of all the fractions. Multiply the whole equation (both sides) by the LCD to get rid of the fractions in an equation.

Examples: Solve for x .

1. $\frac{1}{2} x-\frac{1}{4}=\frac{1}{6}$
2. $\frac{4}{5} x-2=\frac{1}{3}$

Practice: Solve for x .

$$
\text { 1. } \frac{3}{5} x+\frac{1}{5}=\frac{7}{10}
$$

2. $\frac{1}{3} x+\frac{2}{9}=7$

Mixed Review: Solve for x .

$$
\begin{array}{ll}
\text { 1. } 0.5 x+0.9=0.2 x & \text { 2. } \frac{1}{7} x+\frac{3}{7}=8 \\
\text { 3. } \frac{1}{3} x+4=\frac{1}{5} x & \text { 4. } .25 x=0.81-.02 x
\end{array}
$$

Solving Equations Review: Solve each for x. Check each solution after you have solved the problem. If you check a problem and it is wrong, fix it.

1. $x-7=-2$
check:
2. $2 x-3=15$
check:
3. $5=\frac{x}{2}-4$
check:
4. $-6 x=2(x-5) \quad$ check:
5. $\frac{x-7}{5}=9$
check:
6. $\sqrt{x}-3=4 \quad$ check:

Solving Equations Review: Solve each for $x$. Check each solution after you have solved the problem. If you check a problem and it is wrong, fix it.
7. $3(x-7)=-6$
check:
8. $2 x-2=5 x+11 \quad$ check:
9. $\frac{5}{6}=\frac{1}{6} x-2 \quad$ check:
10. $1.3=0.2 x-0.5 \quad$ check:
11. $\frac{1}{4} x+\frac{1}{8}=\frac{1}{2}$
check:
12. $\frac{2 \sqrt{x}-11}{5}=3 \quad$ check:

## I nstructions:

Each of the equations in this envelope has a solution. Solve each equation for x . Arrange the solutions from least to greatest (based on their solution). If you do this correctly, the letters will form a question. Raise your hand when you know the answer to the question that is spelled. Some letters appear more than once.
.. $x-3=-7$
A. $2 \sqrt{x}=6$
N. $\sqrt{3+x}=13$

1. $\frac{x}{2}-9=5$
R. $\frac{5+x}{3}=6$
е. $x=\sqrt{2,500}$
E. $\frac{2}{5} x=\frac{6}{7}$
o. $\frac{x}{3}-1=5$
N. $2 x-9=5 x$
E. $4 x-7=3 x+6$
s. $5=2 x-7$
u. $0.4 x-2=1.2$
o. $x=\sqrt{225}$
н. $4 x=\frac{3}{5}$
s. $0.3 x-3=5.1$
E. $\sqrt{x}-3=6$
т. $2 \sqrt{5+x}=14$
R. $\frac{x-1}{2}=5$
т. $\frac{2}{3} x+\frac{5}{6}=\frac{1}{3}$
Q. $x=\sqrt{0.49}$
D. $10-5 x=15$
x. $4+\frac{x}{3}=15$
o. $2 x-9=5+x$
F. $3 x-11=5 x+7$
F. $5=\frac{1}{2} x-7$
т. $0.5 x-1.1=7.4$
2. $3 x=-9$
3. 
4. $5=13-2 x$
5. $\frac{2}{5} x=4$
6. $\qquad$
7. $\qquad$
8. $\frac{x-3}{5}=2$
9. 
10. $7=\frac{x}{3}-5$
11. $\qquad$
12. $5+x=2 x$
13. $\qquad$
14. $7 x+4=2(x-3)$
15. $\frac{5 x-1}{2}=2 x$
16. 
17. $\qquad$
18. 
19. $\frac{1}{2} x-2=-\frac{1}{5}$
20. 
21. $2.5+1.2 x=4.9$
$\qquad$

Find the square root of each:
11. $\sqrt{49}$
11.
12. $\sqrt{400}$
13. $\sqrt{\frac{9}{16}}$

## Solve for x :

14. $\sqrt{x}=13$
15. $\qquad$
16. $7=\sqrt{x}-5$
17. $\sqrt{3 x}=6$
18. 
19. $5=2 \sqrt{x}-3$
20. 

## Estimate to the nearest whole number:

18. $\sqrt{15}$
19. $\qquad$
20. $\sqrt{85}$
21. 
22. What is the approximate edge length of a square whose area is $200 \mathrm{~cm}^{2}$ ?
(Rounded to the nearest centimeter).
23. $\qquad$


## Solve Inequalities just as you would Equations.

There is ONE DIFFERENCE you must remember:
Whenever you MULTIPLY or DIVIDE both sides by a negative, REVERSE THE DIRECTION OF THE > or <.

## Examples:

1. $4 x-6>-10$
2. $-\frac{2}{3} x \leq 6$
3. $\frac{2-4 x}{3}>6$

## Graphing Your Solutions:



Practice: Solve and graph each.

$$
\text { 1. }-4 x-3>-11 \quad \text { 2. } \frac{-5 x+1}{3} \leq 7 \quad \text { 3. } 2(12-3 x)>-6
$$

Remember: Only switch the direction of the < or > when you multiply or divide by a negative.

Practice: Check each answer by plugging it in (substitution).

1. $3-x>-2$
2. $\frac{-3 x+5}{2} \leq 13$
3. $x-3(2 x-7) \leq 11$
$\qquad$

Solve each. Reverse the direction of the inequality symbol only when you multiply or divide both sides by a negative.
Graph your solution on the line provided.
Use an open circle for < and >.
Use a closed circle for $\leq$ and $\geq$.
If $x$ is greater, the arrow should go to the right.
If $x$ is less, the arrow should go to the right.

1. $-3 x-18>-6$
2. $3 x-2 \leq 4 x+11$
graph: $\qquad$ graph: $\qquad$
3. $\frac{x-1}{2}>-4$
4. $3(x-2) \leq 12$
graph: $\qquad$ graph: $\qquad$
5. $x-2>-6 x+12$
6. $\frac{3}{4} x-2 \leq 7$
graph: $\qquad$ graph: $\qquad$
7. $\frac{7-2 x}{4}>2$
8. $3(x-2) \geq 5(x+1)$
$\qquad$
$\qquad$

Practice:
Determine whether each pair of fractions are equal by making their denominators equal. Fill the blank with $a>,<$ or $=$.

1. $\frac{4}{5}-\frac{5}{7}$
2. $\frac{6}{7}-\frac{7}{9}$
3. $\frac{8}{12}-\frac{6}{9}$

If two fractions are congruent, their cross products will always be equal.

## Examples:

In the following proportions, use cross-products to solve for the variable. Simplify fractional answers.

1. $\frac{x}{15}=\frac{2}{5}$
2. $\frac{2 x}{3}=\frac{3}{4}$
3. $\frac{x-1}{4}=\frac{1}{3}$

Practice: Solve for $x$.

1. $\frac{1}{9}=\frac{x}{4}$
2. $\frac{5}{7}=\frac{3 x}{2}$
3. $\frac{2+x}{7}=\frac{x-4}{5}$

Example: Solve for $x$.

1. $\frac{2}{x-9}=\frac{5}{3 x-1}$

More Difficult Practice. Solve for x . Simplify fractional answers.

1. $\frac{22}{x}=\frac{15}{x-1}$
2. $\frac{5}{7-2 x}=\frac{10}{-4 x-1}$
3. $4=\frac{x}{3 x-1}$
4. $\frac{7}{x-4}=2$

## 

Solve for $\mathbf{x}$. Show all your work. Use Cross-Products and simplify fractional answers.

1. $\frac{3}{9}=\frac{x}{15}$
2. $\frac{3 x}{8}=\frac{9}{12}$
3. $\frac{-5}{x}=\frac{1}{3}$
4. $\frac{10}{7}=\frac{5 x}{14}$
5. $\frac{5}{9}=\frac{10}{x}$
6. $\frac{3 x}{10}=\frac{12}{5}$
7. $\frac{-5 x}{15}=\frac{1}{9}$
8. $\frac{-6 x+1}{12}=\frac{3}{2}$
9. $5=\frac{3 x+1}{5}$
10. $\frac{14}{x+10}=-2$
11. $\frac{2}{5}=\frac{-3 x-3}{30}$
12. $\frac{13}{4}=\frac{2 x-15}{x}$

Write an equation, then solve. Show all your work.
14. The quotient of a number and 28 is $3 / 4$. What is the number?
15. Thirty divided by 4 is equal to the quotient of a number and eight. What is the number?
16. 24 divided by a number is equal to two-thirds. What is the number?
17. The product of 31 and $x$ is equal to the sum of $x$ and 150 . Solve for $x$.

By now, you should be able to recognize the steps to take toward solving simple equations.

## Examples:

What steps (in the correct order) would you take to solve for $x$ ? Do not solve.

1. $\frac{-2 x+8}{5}=-91$
2. $\frac{x-31}{9}+13=-38$
a. Multiply both sides by $\qquad$ .
a. $\qquad$
b. Subtract $\qquad$ .
b. $\qquad$
c. Divide by $\qquad$ .
c. $\qquad$

You can apply similar steps to solve formulas.

## Examples:

What steps (in the correct order) would you take to solve for x ?

1. $\frac{a x+y}{b}=c$
2. $\frac{x-a}{c}+b=y$
a. Multiply both sides by $\qquad$ .
a. $\qquad$
b. Subtract $\qquad$ .
b. $\qquad$
c. Divide by $\qquad$ c. $\qquad$

Practice: Solve for x .

1. $a x-b=c$
2. $\frac{x}{a}-c=f$
3. $\frac{a-x}{w}=v$
4. $\frac{b+x}{c d}-a=y$

Solve for $\mathbf{x}$ ．Show all your work．
1．$x+a=b$
2．$a x+c=b$

3．$b+\frac{x}{c}=y$
4．$a(x+d)=f$

5．$a-x=d$
6．$\frac{x+c}{a}=e$

7．$a=\sqrt{b+x}$
8．$\frac{x}{d}=\frac{b}{c}$

9．$x+y=a-3 x$
10．$a x=b+c$

11．$\frac{x}{f}+b=g$
12．$\frac{a}{b}=\frac{b}{x}$

13．$\frac{d x+a}{f}=g$
14．$x(a+c)=b$

## Instructions:

Each of the equations in this envelope has a solution. Match each numbered problem with the correct solution (the solutions have letters). If you get htem all right, they will spell-out a question. Raise your hand when your team has the answer to that question. Note: There are more solutions than problems. Some solutions will not be used.

1. $\sqrt{x-3}=7$
s. $x=52$
2. $5 x-9=7 x-13$
o. $x=2$
u. $x=-2$
3. $\frac{5-x}{3}<6$
L. $x>-13$
в. $x>13$
4. $\frac{x-a}{c}=b$
v. $x=b c+a$
т. $x=c(b+a)$
5. $5 x=\frac{7 x-9}{2}$
е. $x=-3$
6. $\frac{\sqrt{x+1}}{2}=6$

ง. $x=143$
A. $x=145$
7. $\frac{x+a y}{d}=c$
u. $x=c d-a y$
8. $-2 x-11>15$
м. $x<-13$
т. $x<13$
9. $f x-a=b$
в. $x=\frac{b+a}{f}$
10. $5 x-9<11$
E. $x<4$
R. $x=12$
o. $x=3$
11. $\frac{x}{3}+2=6$
18. $5 x+9=7 x+3$
12. $\frac{f x}{a}=b$
E. $x=27$
19. $0.6 x-4.1=1.3$
13. $\frac{x}{3}=\frac{x-9}{2}$
s. $x=\frac{a b}{f}$
R. $x=9$
14. $\frac{\sqrt{x+2}}{3}=5$
v. $x=223$
20. $-9<2 x-17$
15. $\frac{x-3}{2}-1=5$
E. $x=15$

צ. $x>4$
16. $-3 x+5=2$
N. $x=1$
?. $x=-1$
17. $\frac{x-6}{2}=5 x$
F. $x=-\frac{2}{3}$

ต. $x=-13$

Solve for $x$ :

1. $3-x=5$
2. $x=$
3. $x=$
4. $x=$
5. $x=$
6. $x=$
7. $x=$
8. $x=$
9. $x=$
10. $x=$


Solve forx: Solve the following INEQUALITIES.
WRITEAND GRAPH YOUR ANSWER. ex: $\underline{\mathbf{x}>-\mathbf{2}}$

10. $\frac{x-3}{2}>-5$
10. $\qquad$
11. $-3 x+7<-5$
11. $\qquad$
$\qquad$
12. $-3 \leq \frac{5-x}{2}$
12. $\qquad$

Solve each formula for $\mathbf{x}$ :
13. $\frac{x-a}{b}=c$
13. $x=$ $\qquad$
14. $\frac{x}{a}+b=d$
$\qquad$

## Write an equation to solve each:

15. Three less than five times a number is 12. What is the number?
16. $x=$ $\qquad$
17. Two times the sum of a number and 11 is 32 . What is the number?
18. $x=$ $\qquad$
19. The number of pigs on a farm is three less than twice the number of horses. If there are 17 pigs on the farm, how many horses are there?
