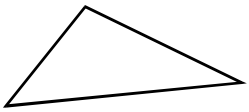
Polygon Interior/Exterior Angles

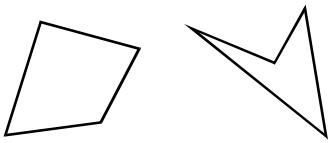
Geometry 5.1

Recall the sum of a polygon's interior angles:

Sum of interior angles in any triangle: _____



Sum of interior angles in any quadrilateral: _____

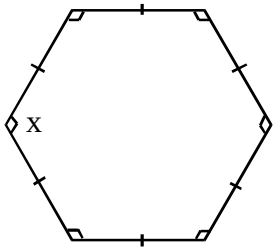


Sum of the angles in any n-sided figure: _____

Practice:

Find the missing angles in each figuure below:

1.



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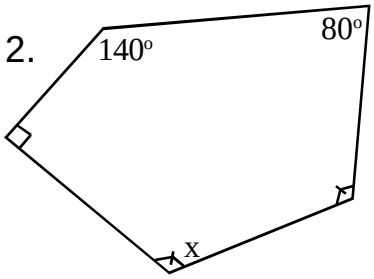
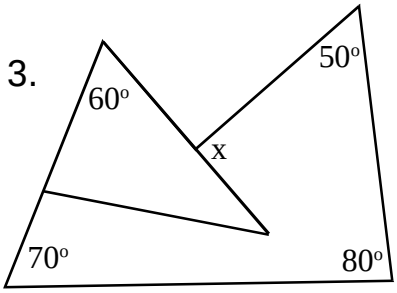
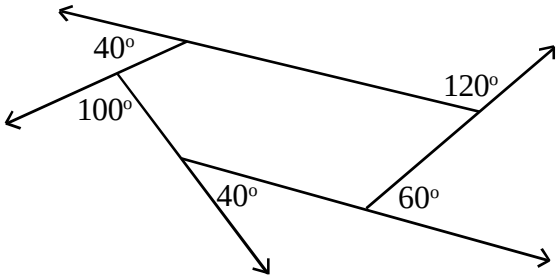


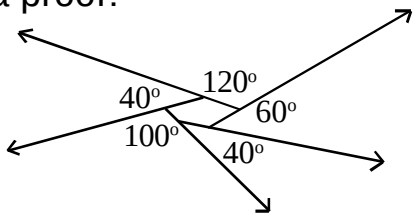
3.

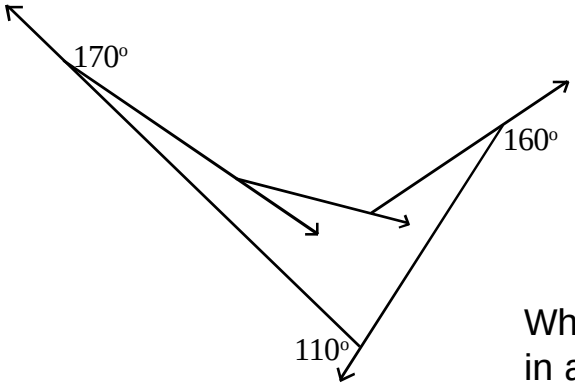


Exterior angles:



The sum of the exterior angles in a polygon always equals 360°. This is easier to explain with a diagram than a proof:





Concave polygons:

An easy way to define an exterior angle is: The supplement of the corresponding interior angle.

What must be true about the exterior angles in a convex figure?

What is the sum of the two missing exterior angles in the figure above?

Exterior Angles

Geometry 5.2

Answer:

1. The exterior angles are **supplementary** to the interior angles in a regular polygon with n sides.
360/n gives you the measure of each exterior angle in a regular n-gon.
Therefore: **n[180-(360/n)]** can be used to find the sum of all angles in a regular n-gon. Is this the same as 180(n-2)?

2. What is the exterior angle measure of a 15-gon?

3. In regular decagon ABCDEFGHIJ, what is the measure of angle ACB?

4. The interior angles of a polygon measure 179^o. How many sides does the figure have?

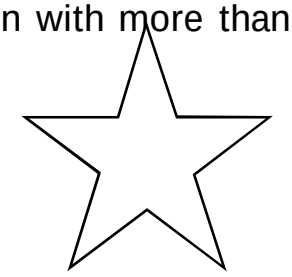
5. A regular polygon has exterior angles measuring 12^o.
How many sides does the polygon have?

Answer:

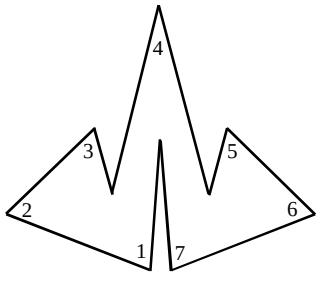
1. Is it possible for a triangle to have an acute exterior angle?

2. A polygon has 11 sides and 10 congruent 150^o interior angles.
What are the measures of its exterior angles?

3. The star below has ten sides and five acute interior angles. Can you draw a ten-sided polygon with more than five acute interior angles?



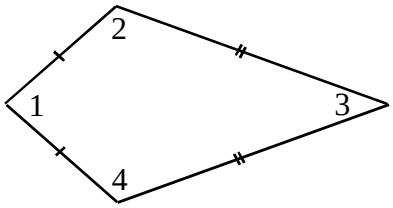
8 is not possible because $8(90)=720^o$
leaving 720^o for the remaining 2 angles.



Kites and Trapezoids

Geometry 5.3

Parts of a Kite:



∠1 and ∠3 are Vertex Angles

∠2 and ∠4 are Nonvertex Angles

More proofs!
No need to write them, just explain how each could be done.

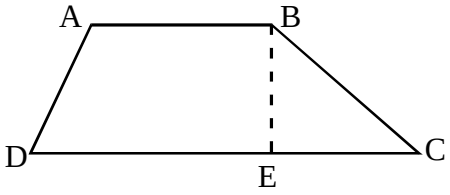
1. How could you prove that the nonvertex angles are congruent?

2. How could you prove that the diagonal connecting the vertex angles bisects them?

3. How could you prove that the two diagonals are perpendicular to each other.

4. How could you prove that the diagonal connecting the vertex angles bisects the other diagonal?

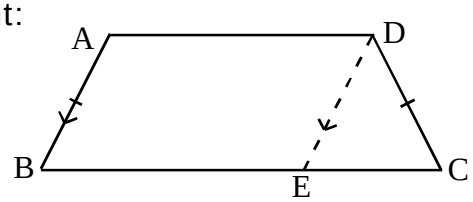
Parts of a Trapezoid:



$\overline{AB}$ and $\overline{CD}$ are bases.
∠A and ∠B are a pair of base angles.
What is the other pair?
If $\overline{AD} \cong \overline{BC}$, the trapezoid is an **Isosceles Trapezoid**
$\overline{BE}$ is an altitude.

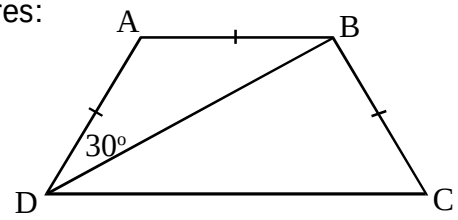
1. How can you prove that consecutive angles of a trapezoid are supplementary?

2. How can you prove that the base angles of an isosceles trapezoid are congruent?
hint:



3. Using the information from #2, how can you prove that the diagonals in an isosceles trapezoid are congruent?

4. Find the angle measures:



m∠ABD =
m∠ABC =
m∠BCD =
m∠CDB =

Midsegments

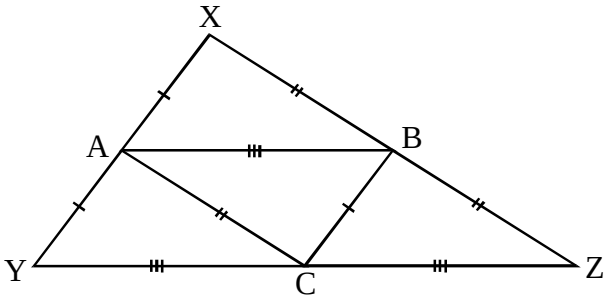
Geometry 5.4

Midsegments of a triangle connect the midpoints of the sides.
Every triangle has three midsegments.

Discover properties of triangle midsegments by completing the following construction:

1. Draw a hand-sized scalene triangle on your paper.
2. Construct and connect the three triangle midsegments.

You should have created four triangles.
Measure the sides of all four triangles.
What do you notice?

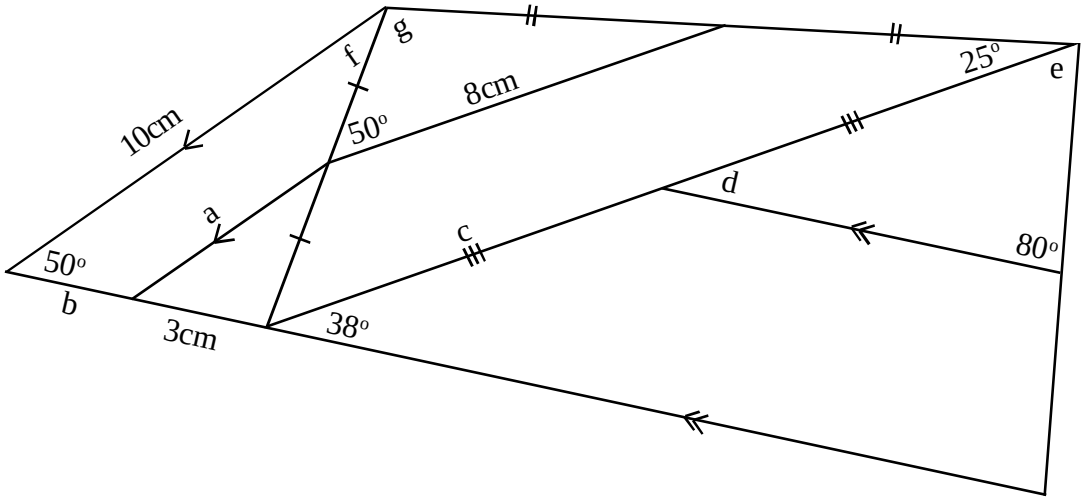


Complete the following statements about midsegment AB:

Segments AB and YZ are _____. Explain the proof (use congruent △s)

Segment AB is _____ the length of segment YZ. Explain the proof.

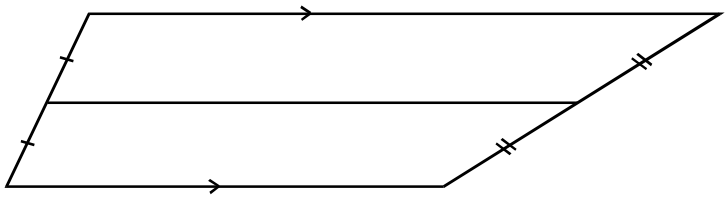
Determine the measure of each missing side or angle below:



Midsegments

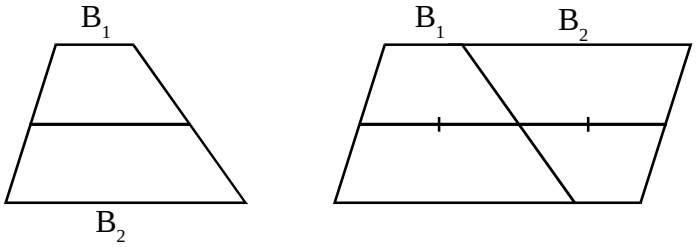
Geometry 5.4

Midsegments of a trapezoid connect midpoints of the non parallel sides.



The midsegment of a trapezoid will always be parallel to the bases. The simplest proof that the midsegment is parallel to the bases involves coordinate geometry and slope.

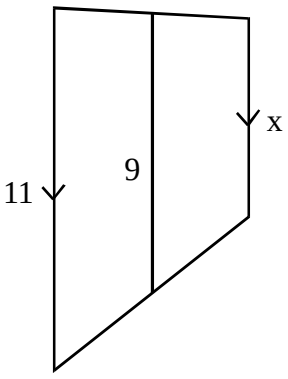
The length of the midsegment is related to its bases as well.



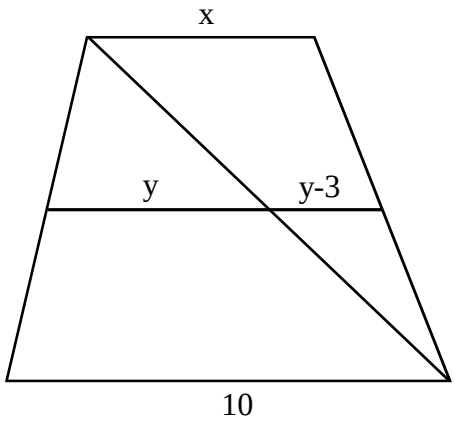
Can you write a formula for the length of the midsegment of a trapezoid using B_1 and B_2 as the bases?

Solve:
Find length x given the following midsegments:

1.



2.



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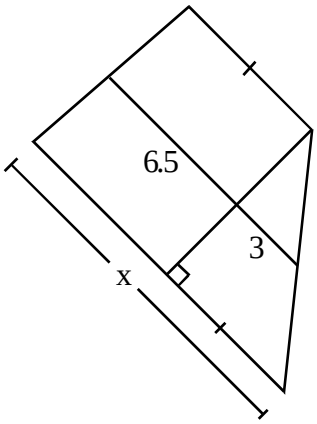
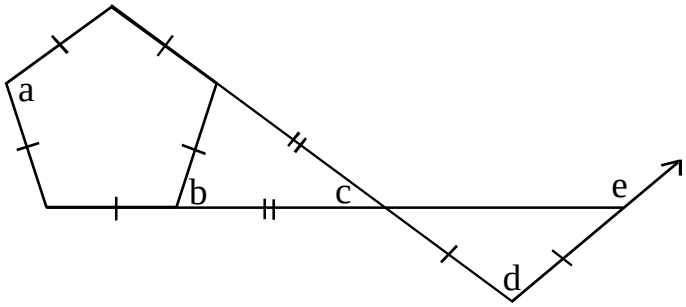


Review

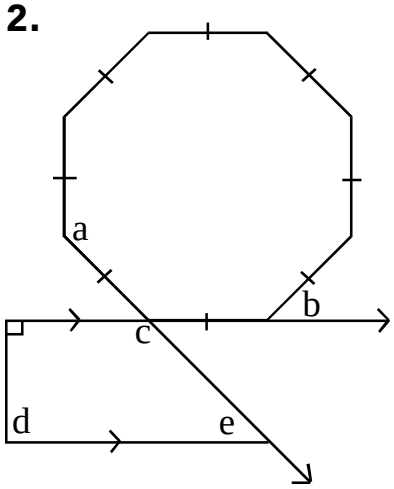
Geometry 5.4

Determine the measure of each angle labeled below:

1.



2.

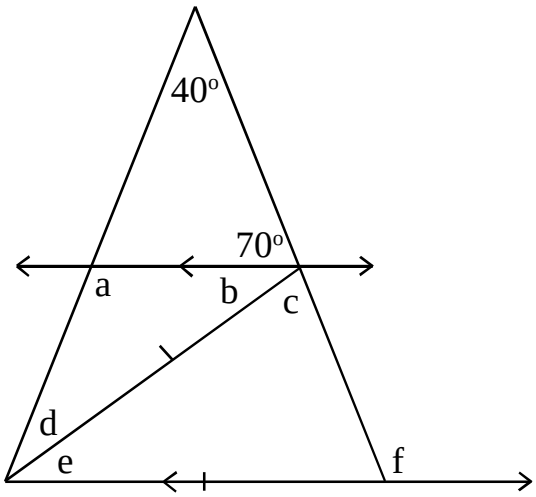


3. What is the maximum number of obtuse exterior angles in a hexagon?

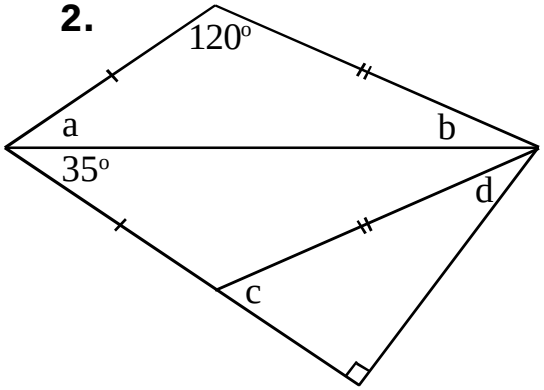
4. Determine the measure of the interior angles in a regular 36-gon.

Determine the measure of each angle labeled below:

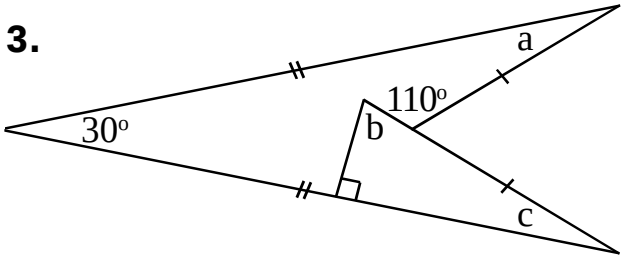
1.



2.



3.



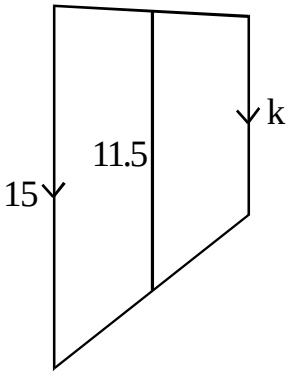
4. Two angles in a kite measure 50 and 110 degrees. List all possible combinations of the other two angles.

Review

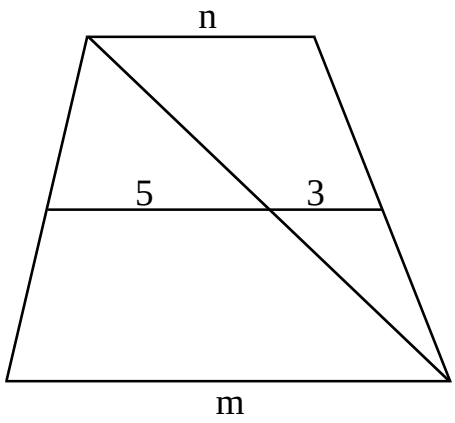
Geometry 5.4

Determine the measure of each missing side below in each trapezoid. Midsegments are drawn in each.

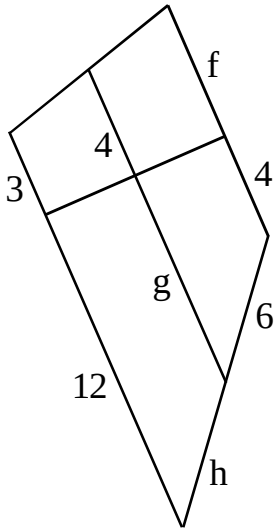
1.



2.

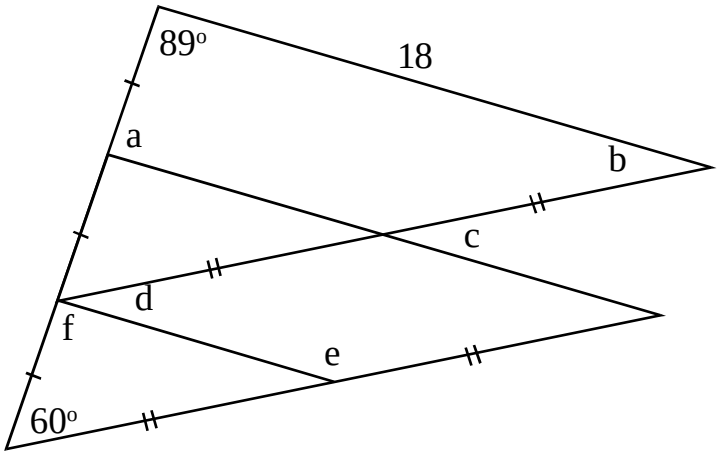


3.

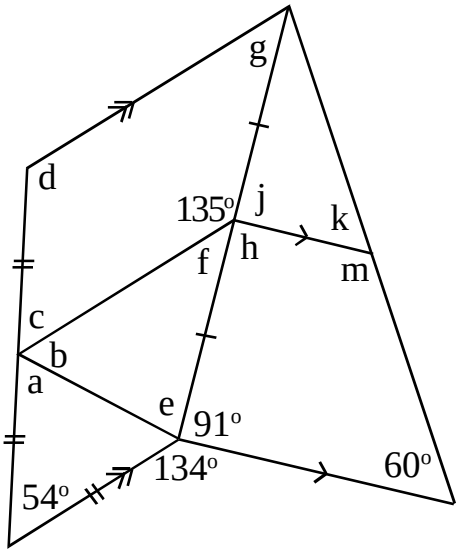


Determine the angle measure for each:
Write 'cannot be determined' where applicable.

1.



2.



Parallelogram Properties

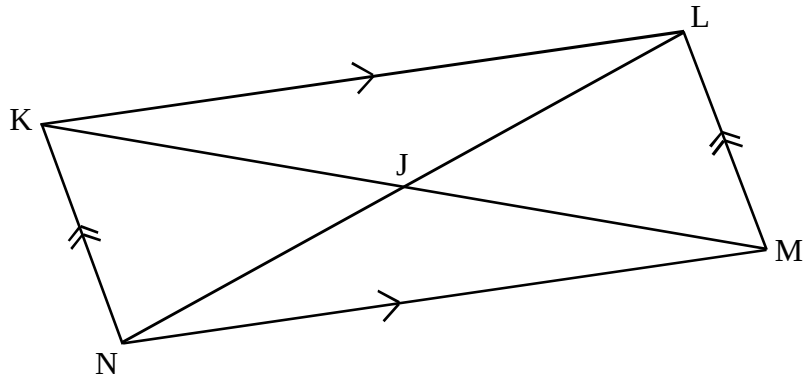
Geometry 5.5

Using a straight edge, draw **parallelogram KLMN** that is **not a rectangle or a rhombus.**

Complete the following statements about the parallelogram:

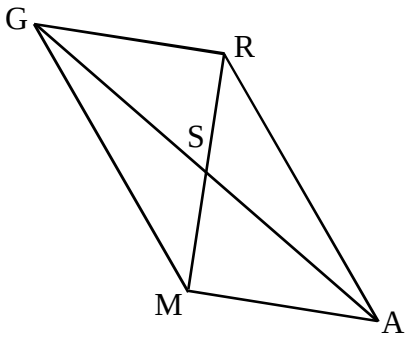
Opposite angles are _____.
Consecutive angles are _____.
Opposite sides are _____.
Diagonals of the parallelogram _____ _____ _____.

Explain the proof of each statement above using the figure below:

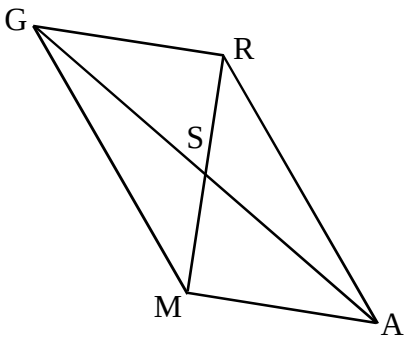


Solve:

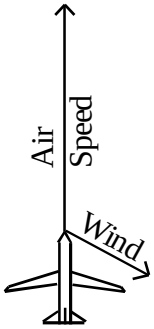
1. Given:
RS = 10cm
MA = 16cm
GM = 21cm
Find the perimeter of △GRM.



2. Given:
Perimeter of △GRS=21
Perimeter of △RAS=26
Perimeter of △GRA=35
Find the length of $\overline{RM}$.



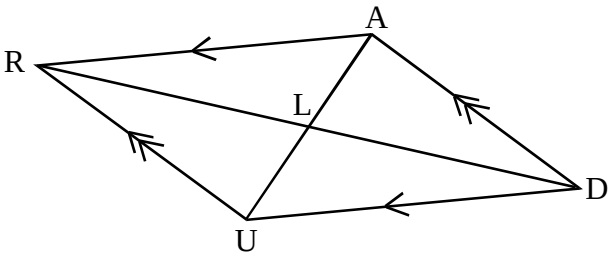
3. Vectors:
A plane flies north at 250mph, with a headwind as shown blowing 40mph. draw a line to represent the path and speed of the plane?



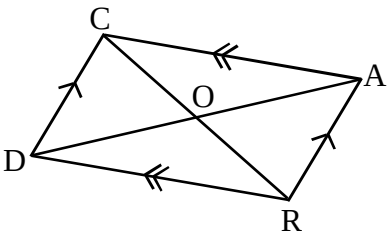
Perimeter Problems

Geometry 5.5

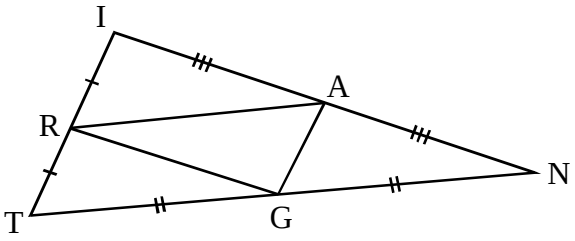
Solve: **1.** Given:
Perimeter of △RAD=24cm
Perimeter of △RAU=21cm
$\overline{AU}$=7cm
Find the length of $\overline{RD}$.



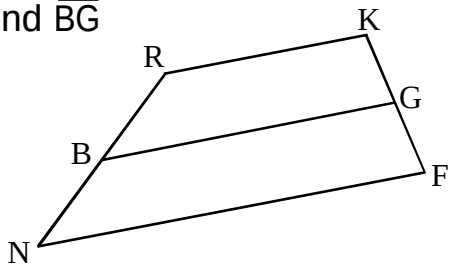
2. Given:
Perimeter of △DOC=18cm
Perimeter of △CAR=26cm
$\overline{CO}$=5cm $\overline{CA}$=9cm
Find the length of $\overline{DO}$.



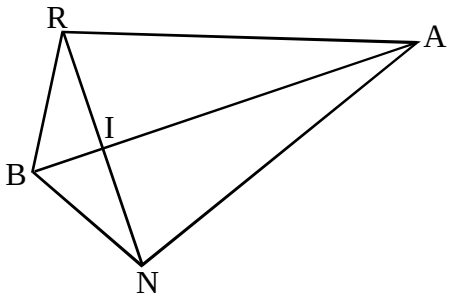
Solve: **1.** Given:
Perimeter of △TIN=24cm
Find the perimeter of △RAG



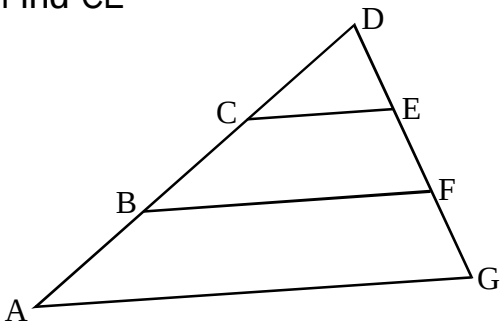
2. Given:
$\overline{GB}$ is a midesgment of trapezoid NRKF.
Perimeter of BRKG = 31m
Perimeter of BGFN = 37m
$\overline{RK}$ = 10m
Find $\overline{BG}$



3. Given:
△RIA is a 5-12-13 right triangle
Perimeter of △BRN = 22km
Find the perimeter of kite BRAN



Challenge:
$\overline{CE}$ is the midsegment of △BDF
$\overline{BF}$ is the midsegment of ACEG
The perimeter of △CDE is 17
The perimeter of ACEG is 46
Find CE



Name_____ Period _____

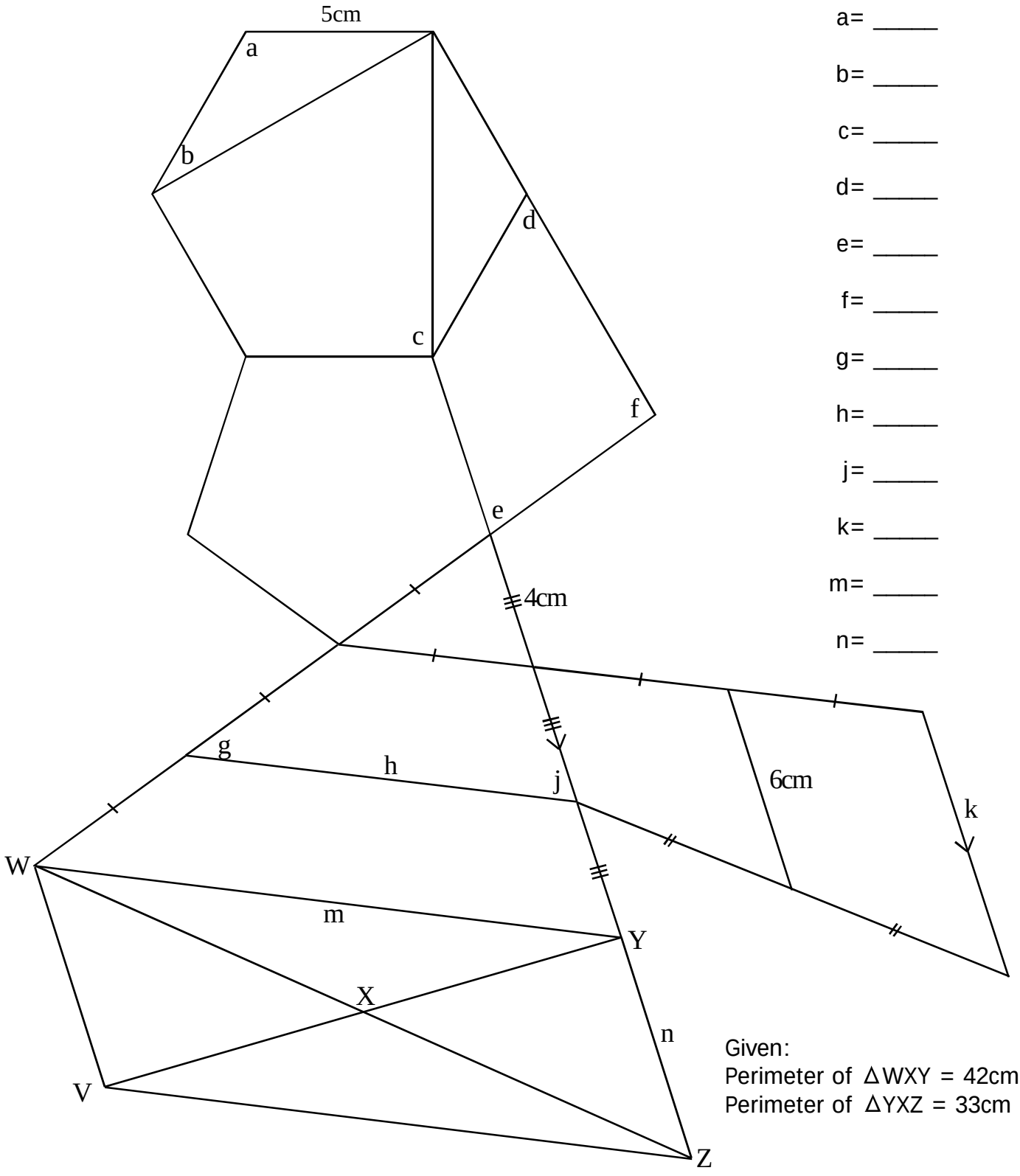
Polygon Properties Review

Geometry 5.5

Determine the measures of each missing angle or side labeled below:

Not To Scale

The hexagon and pentagon are both regular.



a= _____

b= _____

c= _____

d= _____

e= _____

f= _____

g= _____

h= _____

j= _____

k= _____

m= _____

n= _____

Given:

Perimeter of ΔWXY = 42cm

Perimeter of ΔYXZ = 33cm

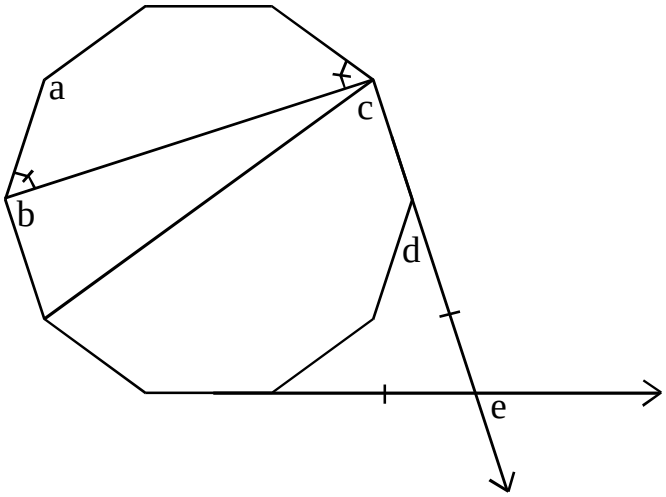
Name_____ Period _____

Polygon Properties Practice

Geometry 5.5

Determine the measures of each missing angle below:

The decagon in the figure is regular.



1. a= _____

2. b= _____

3. c= _____

4. d= _____

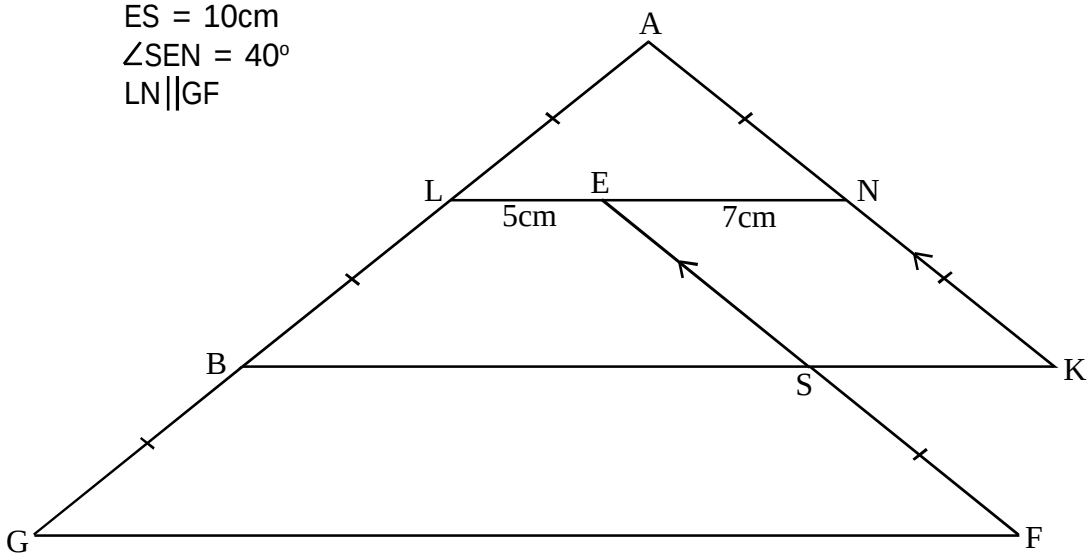
5. e= _____

6. Two of the angles in a convex kite measure 105° and 145°. What is the SMALLEST possible angle measure in the kite?

6. _____

Determine the measures or length for each:

Given:
ES = 10cm
∠SEN = 40°
LN||GF



7. ∠A= _____

8. $\overline{BK}$= _____

9. ∠ALN= _____

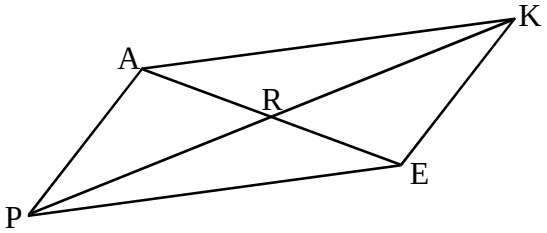
10. $\overline{SK}$= _____

11. $\overline{NK}$= _____

12. ∠BSF= _____

13. $\overline{GF}$= _____

14. Use the given information to find the perimeter of parallelogram PAKE:



Given:
Perimeter of ΔARK = 16cm
Perimeter of ΔKER = 13cm
AK = 7cm

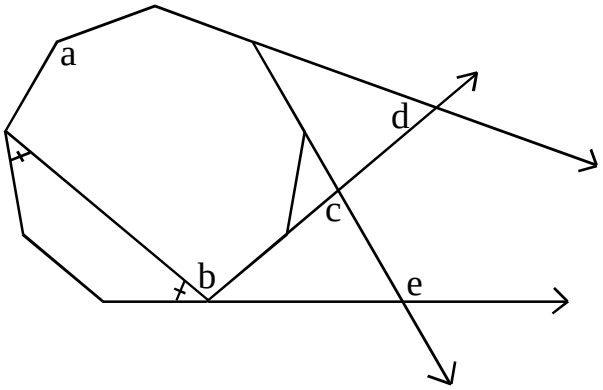
14. _____

Name_____ Period _____

Polygon Properties Practice Quiz Geometry 5.5

Determine the measures of each missing angle below:

The nonagon in the figure is regular.



1. a= _____

2. b= _____

3. c= _____

4. d= _____

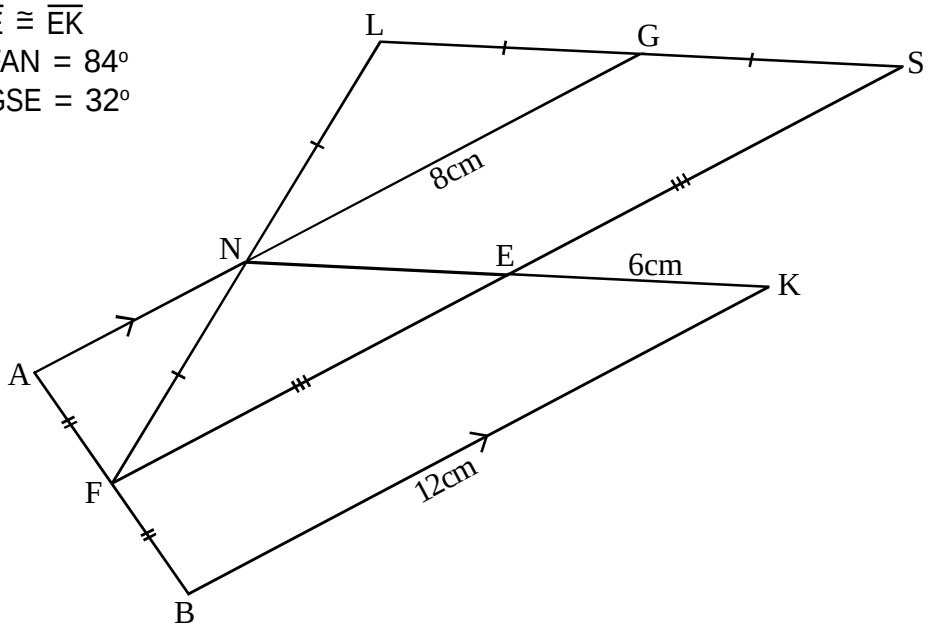
5. e= _____

6. Two of the angles in a convex kite measure 100° and 45°. What is the LARGEST possible angle measure in the kite?

6. _____

Determine the measures or length for each: (not to scale)

Given:
$\overline{NE} \cong \overline{EK}$
$\angle FAN = 84^\circ$
$\angle GSE = 32^\circ$



7. ∠L= _____

8. $\overline{ES}$= _____

9. ∠NES= _____

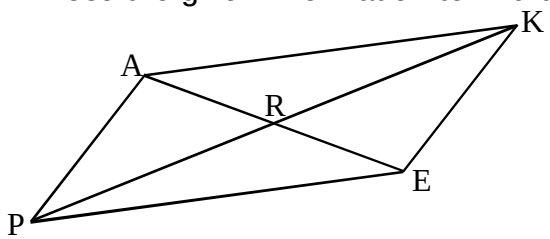
10. $\overline{LS}$= _____

11. $\overline{AN}$= _____

12. ∠FBK= _____

13. $\overline{FS}$= _____

14. Use the given information to find the length of AE:



Given:
Perimeter of ΔAKE = 33cm
Perimeter of PAKE = 42cm

14. _____

Name_____ Period _____

More Difficult Polygons Practice

Geometry

Solve each:

1. In regular octagon ABCDEFGH, what is the measure of angle ACH?

_____

2. In parallelogram ABCD, Diagonal AC is twice the length of diagonal BD. The perimeter of triangle ABC is 21 and the perimeter of triangle BCD is 17cm, what is the perimeter of parallelogram ABCD?

_____

3. Convex kite QRST has vertex angles Q and S, where Q is three times the measure of S. Angle QRT is 30 degrees, find the measure of angle SRT.

_____

4. The sides of a regular polygon are 2cm long and the interior angles of the polygon measure 171 degrees. What is the perimeter of the polygon?

_____

5. The midpoints of quadrilateral ABCD are connected to form quadrilateral WXYZ. If diagonals AC=10cm and BD=8cm, what is the perimeter of quadrilateral WXYZ?

_____

6. What is the smallest angle which can be created by connecting three vertices of a regular 36-gon?

_____

7. The long diagonal of a rhombus is 3 times the length of the short diagonal. The perimeter of the rhombus is 40cm, what is its area? (Hard, think it through!)

_____

Challenge: How many regular polygons have interior angle measures of integral (integer) measure?

_____

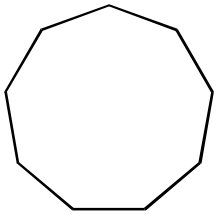
Name_____ Period _____

More Dificult Polygons Practice

Geometry

Solve each:

1. In regular nonagon ABCDEFGHI, what is the measure of angle AFB?



_____

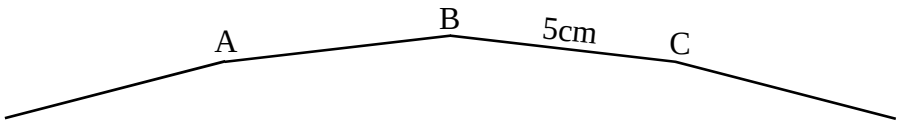
2. Rhombus ABCD has diagonal lengths of 10 and 24.
What is the perimeter of the rhombus?

_____

3. Concave kite FGHJ has a vertex angle H measuring 10 degrees.
Interior angle F is three times the measure of angle G. What is the measure of the non-vertex angles?

_____

4. In the section of regular polygon below, the measure of angle ACB is 5 degrees.
What is the perimeter of the polygon?



_____

5. The midpoints of quadrilateral WXYZ are connected to form *rhombus* PQRS.
If the perimeter of PQRS = 18cm, what is the length of diagonal WY?

_____

6. One of the angles in a convex kite measures 174 degrees.
One vertex angle is two degrees larger than the other.
What is the smallest possible angle measure in the kite?

_____

7. The diagonals of parallelogram ABCD intersect at X. If AX = 25, and AB = 7, and angle ABD is a right angle. What is the area of the parallelogram?

_____

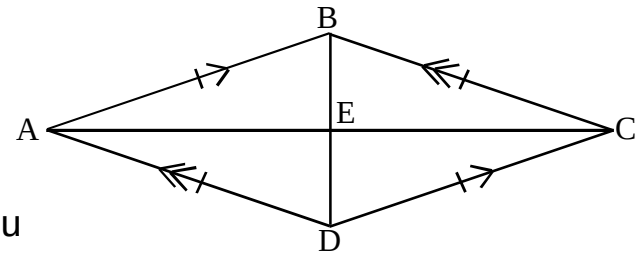
Special Parallelograms

Geometry 5.6

Special Parallelograms:
Rhombuses are equilateral _____.
Rectangles are _____ parallelograms.
Squares are _____ and _____ parallelograms.

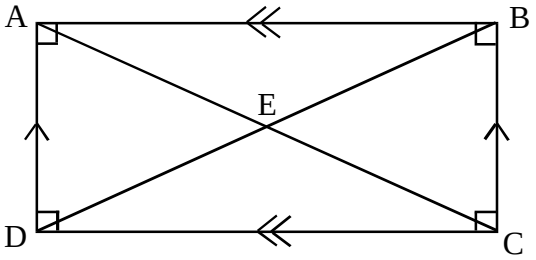
Construct rhombus ABCD on your paper with diagonals AC and BD.
What can you infer about the diagonals of a rhombus?

Are they perpendicular?
Do they bisect each other?
Are they congruent?
Do they bisect the angles?
Explain the proof for each time you answered 'yes' above.



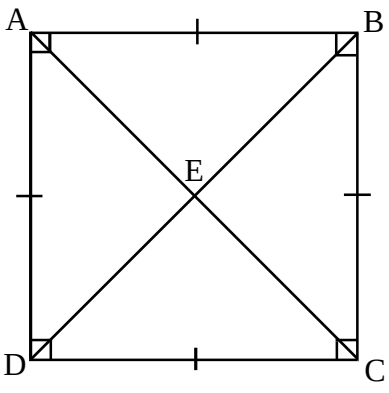
Construct rectangle ABCD on your paper with diagonals AC and BD.
What can you infer about the diagonals of a rectangle?

Are they perpendicular?
Do they bisect each other?
Are they congruent?
Do they bisect the angles?
Explain the proof for each time you answered 'yes' above.



Construct square ABCD on your paper with diagonals AC and BD.
What can you infer about the diagonals of a square?

Are they perpendicular?
Do they bisect each other?
Are they congruent?
Do they bisect the angles?
Explain the proof for each time you answered 'yes' above.

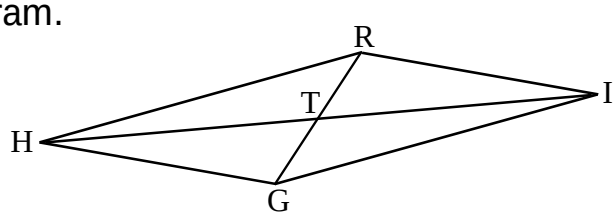


Proofs Practice

Geometry 5.7

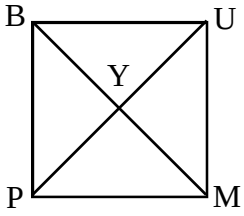
Prove:
Write a two-column or flowchart proof for each of the following:

1. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.



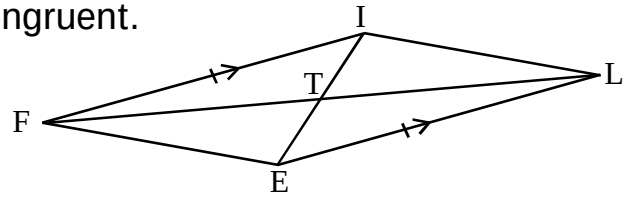
Challenge. If the diagonals of a quadrilateral are perpendicular, congruent, and bisect each other, then the quadrilateral is a square.

hint: Use "Congruent parts of congruent segments" in your proof.



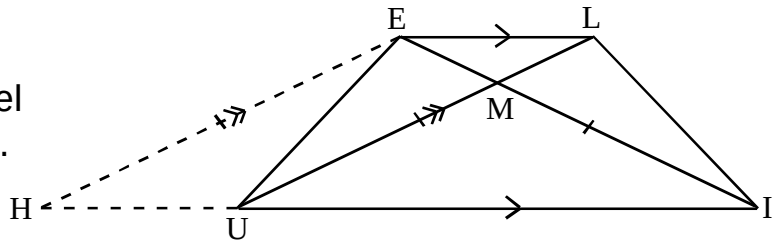
Prove:
Write a two-column or flowchart proof for each of the following:

1. Prove that if two sides of a quadrilateral are congruent and parallel, then the other two sides are congruent.



Challenge. If the diagonals of a trapezoid are congruent, then the trapezoid is isosceles.

hint: Construct the parallel line shown in the diagram.



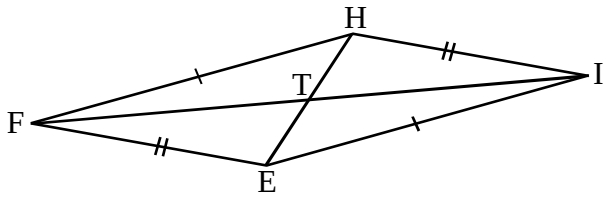
Name_____ Period _____

Proofs Practice

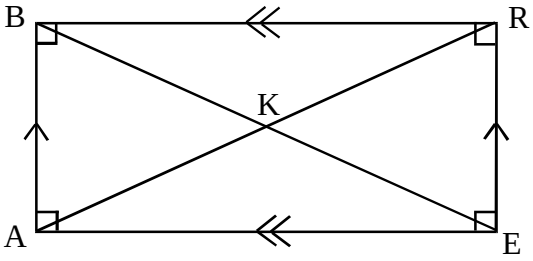
Geometry 5.7

Write a Complete Proof for each statement and diagram below. You may use a two-column or flow-chart proof. Only use the information included in the statements and diagrams.

1. Prove that if opposite sides of a quadrilateral are congruent, then opposite sides are parallel.



2. Prove that the opposite sides of a rectangle are congruent.



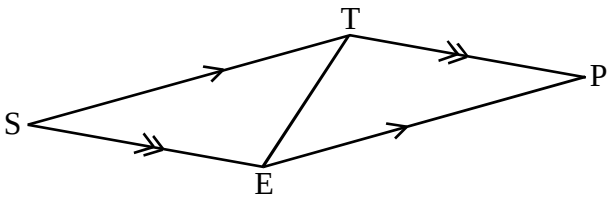
Name_____ Period _____

Proofs Practice

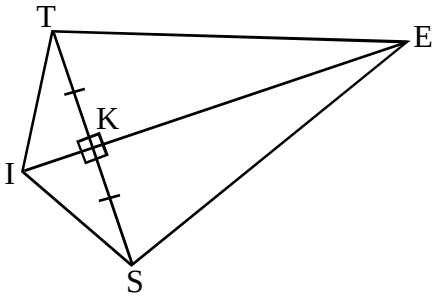
Geometry 5.7

Write a Complete Proof for each statement and diagram below. You may use a two-column or flow-chart proof. Only use the information included in the statements and diagrams.

3. Prove that opposite angles in a parallelogram are congruent.



4. Prove that if the diagonals of a quadrilateral are perpendicular and one bisects the other, then the quadrilateral is a kite.



Name_____ Period _____

Quadrilaterals Practice Test

Geometry 5.7

Answer the following:

1. The measure of an interior angle in a regular octagon is ___.

1. _____

2. If the exterior angles in a regular polygon measure 72 degrees, how many sides does the figure have?

2. _____

3. The sum of the interor angles in a polygon is equal to the sum of the exterior angles. How many sides does the polygon have?

3. _____

4. In an isosceles trapezoid, two of the angles measure 56 degrees. What are the measures of the other two angles?

4. _____

5. The two vertex angles in a kite measure 55 and 170 degrees. What are the measures of the nonvertex angles?

5. _____

6. The perimeter of the triangle formed by the midsegments of triangle ABC is 14 inches. If AB = 6 and BC = 12, find the length of AC.

6. _____

7. Trapezoid EFGH has bases measuring x-10 and 3x. If the midsegment is 19 inches long, what is the length of the short base?

7. _____

8. Triangle ABC has midpoint D on segment AB and E on segment AC. If the perimeter of triangle ADE is 20 inches, what is the perimeter of triangle ABC?

8. _____

9. In parallelogram ABCD, angle A measures $(x+5)^o$ and angle C measures $(2x-16)^o$. Find the measure of angle B.

9. _____

10. In parallelogram QRST, the diagonals intersect at X.
The perimeter of QRST is 26.
The perimeter of RST is 20.
Find the length of RX.

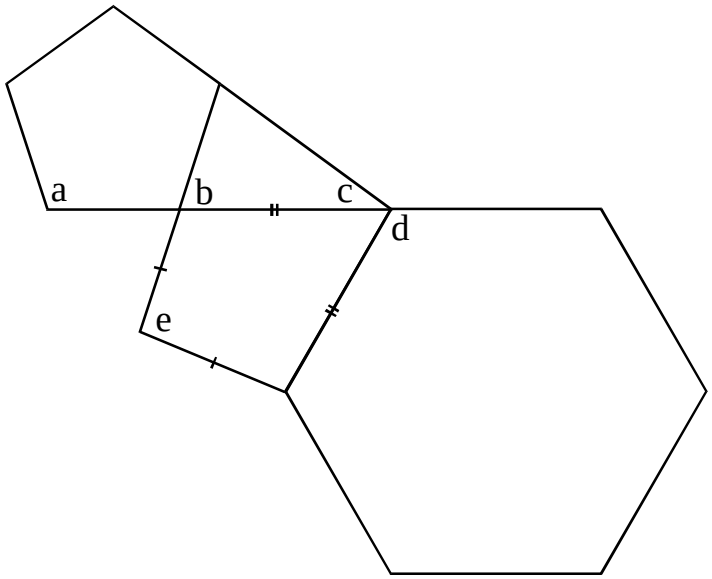
10. _____

Name_____ Period _____

Quadrilaterals Practice Test

Geometry 5.7

Find each missing angle:
The pentagon and hexagon are regular.



11. a=_____

12. b=_____

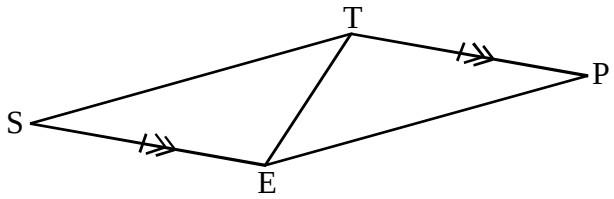
13. c=_____

14. d=_____

15. e=_____

Write a two-column proof to prove that if opposite sides of a quadrilateral are congruent and parallel, then the remaining sides are also parallel.

note:
Use only what is given in the diagram.



_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____

Quadrilaterals Test Review

Geometry 5.7

What is the measure of an interior angle in a regular pentagon?

The exterior angles in a regular polygon measure 18 degrees. How many sides does it have?

One vertex angle in a kite measures 57 degrees and one nonvertex angle measures 125 degrees. What are the measures of the other angles?

An isosceles triangle has a perimeter of 42 inches, and has mid-points A, B, and C. If BC = 3 inches, what is the length of AB?

The midsegment of a trapezoid measures (x+3) inches. The long base of the trapezoid measures (2x+3) inches. What is the length of the short base?

Midsegment GH of triangle XYZ is parallel to side YZ. If GH is 7 inches long, GX is 6 inches, and XZ is 16 inches, what is the perimeter of triangle XYZ?

Rectangle WAVE has diagonals which intersect at N. WAVE has a perimeter of 34 inches. If the perimeter of triangle AVE is 30 inches, what is the length of AN?

Isosceles trapezoid QRST has midsegment AB which is 10 inches long. The perimeter of ARSB is 40 inches and the perimeter of QABT is 46 inches. What is the length of RS?

One of the exterior angles in a rhombus is 37 degrees. What are the measures of the four interior angles?

In parallelogram GRAM, angle G is (3x-5) degrees and angle R is (x+25) degrees. What is the measure of angle A in degrees?
(give the actual angle measure in degrees)

In parallelogram MANY the diagonals intersect at X.
If MX = (2x+7) cm and NX = (3x-5) cm, how many centimeters long is MN? (give the actual length)

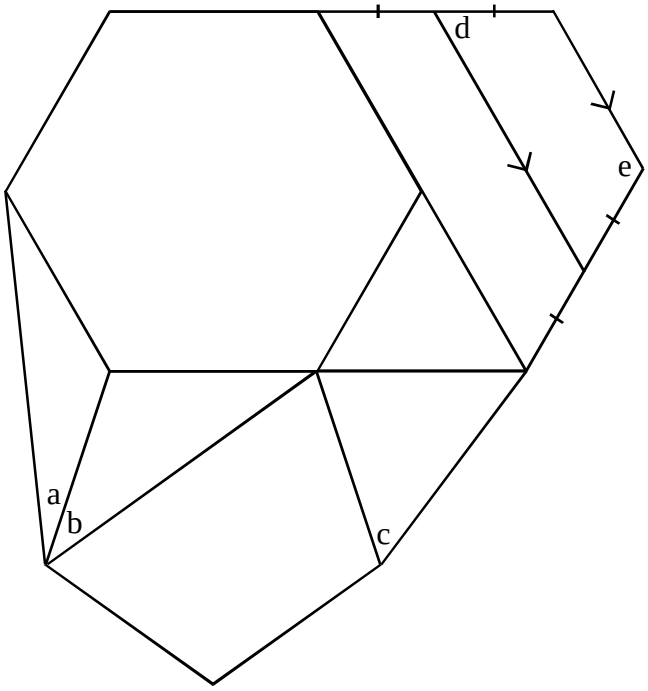
Name_____ Period _____

Polygon Review

Geometry Re

1. Find each missing angle measure below:

Note: All figures that appear regular are regular.



a _____

b _____

c _____

d _____

e _____

2. Two angles of a convex kite measure 120 and 90 degrees. What is the largest possible measure of the other angle(s)?

2. _____

3. Isosceles triangle ABC has a perimeter of 18 inches, with congruent sides AB and BC. AC = 4 inches. Triangle XYZ is formed by the midsegments of triangle ABC. What are the lengths of all three sides of triangle XYZ?

3. ____ ____ ____

4. A trapezoid has midsegment JK. One of the bases of the trapezoid is 12 inches longer than the other. JK is 11 inches long. What is the length of the short base?

4. _____

5. In parallelogram ABCD, angle A measures $(x+24)^o$ and angle B measures $(2x-60)^o$. Find the measure of angle D.

5. _____

6. In parallelogram EXTR the diagonals intersect at A. If EA is (7x-6) cm long and AT is (2x+4) cm long, what is the length of segment ET in centimeters?

6. _____

7. In triangle ART, midsegment KB is parallel to side AT. If angle KBT is 75 degrees, what is the measure of angle ATB?

7. _____

Name_____ Period _____

Polygon Review

Geometry Re

8. Find each missing angle measure below:
Note: The nonagon is regular.

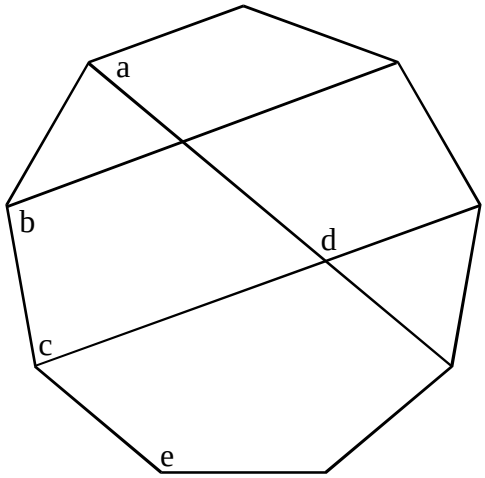
a _____

b _____

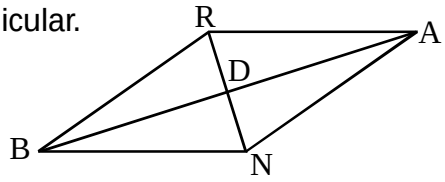
c _____

d _____

e _____



9. Prove that the diagonals of a rhombus are perpendicular.
Note: only use the following given information:
Opposite sides are parallel.
All sides are congruent.



_____ _____

_____ _____

_____ _____

_____ _____

_____ _____

_____ _____

Add more lines on separate paper if needed.

10. Given:
Perimeter of $\Delta MSA = 39$
Perimeter of $\Delta MSG = 44$
Perimeter of $\Delta MGA = 63$

MR = _____