

# Segments and Angles

## Geometry 3.1

All constructions done today will be with **Compass and Straight-Edge ONLY**.

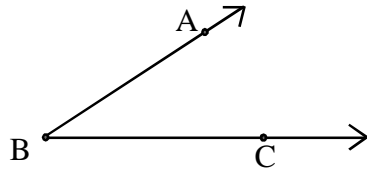
**Duplicating a segment is easy.**

To duplicate the segment below: Draw a light, straight line. Set your compass to the length of the original segment. Use your compass to mark the length of the segment.

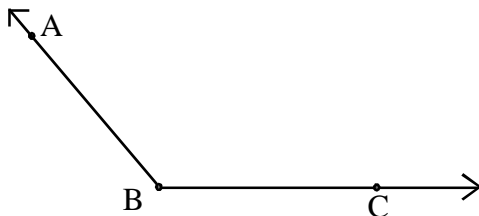
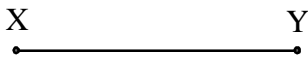


**To duplicate an angle:**

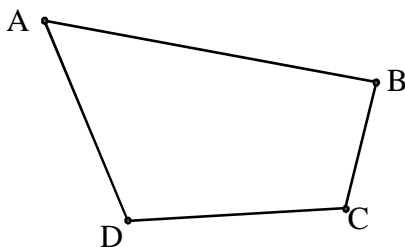
Draw a ray. Draw an arc on the original angle. Draw the same arc on your ray. Now, set the compass equal to the distance between where the arc intersects the angle on the original figure. Duplicate that point on the new figure. Draw a line from the end-point of the ray through the arc. (Confused? Just watch me do it on the board).



**Easy Practice:** Duplicate each segment or angle with only a straight-edge and compass in the space to the right of each.



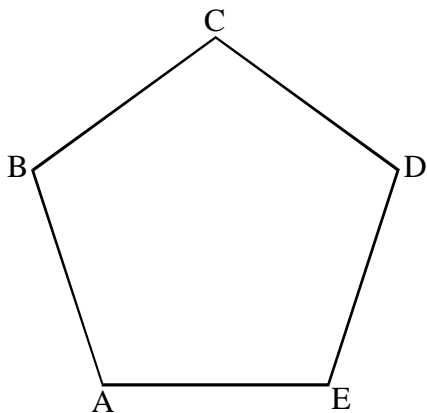
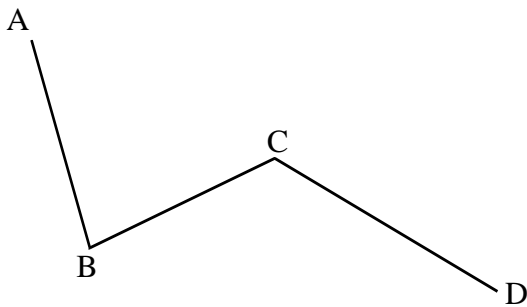
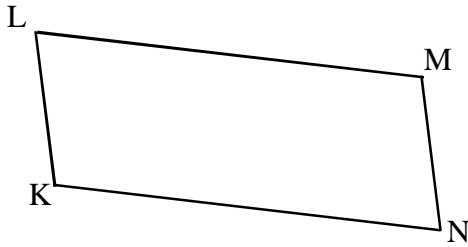
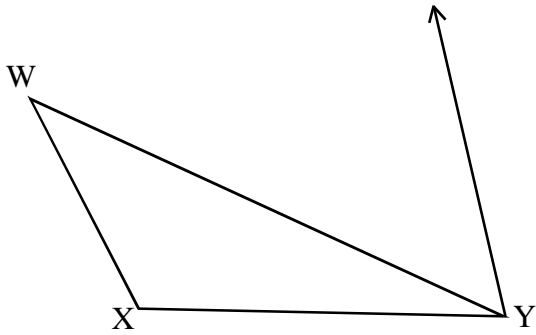
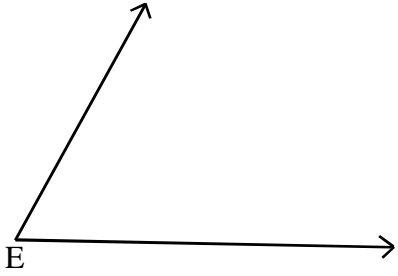
**Challenge:** (It is all lines and angles).



# Segments and Angles

## Geometry 3.1

Use what you have learned to duplicate each of the objects below:

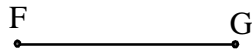


# Segments and Angles

## Geometry 3.2

All constructions done today will be with **Compass and Straight-Edge ONLY**.

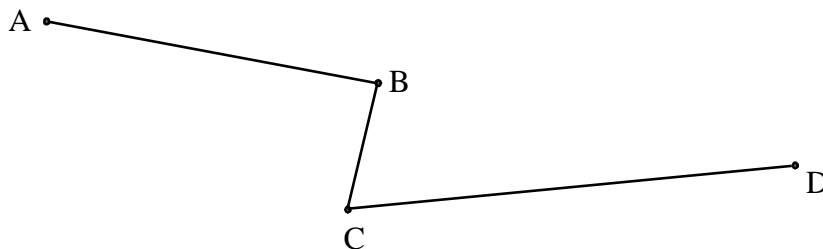
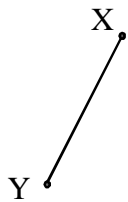
**Constructing a perpendicular bisector:** Follow the steps shown by Mr. Batterson on the board to bisect the line below with a perpendicular.



What is the relationship of points F and G to all points along the perpendicular bisector?

\_\_\_\_\_

Construct a perpendicular bisector for each segment below.



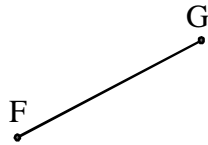
# Segments and Angles

## Geometry 3.2

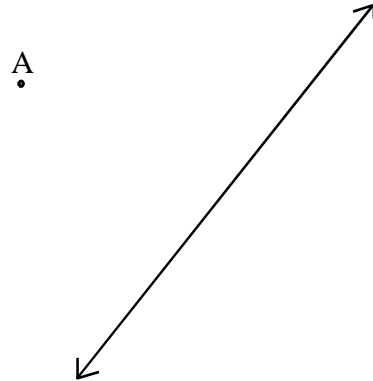
All constructions done today will be with **Compass and Straight-Edge ONLY**.

Constructing a perpendicular bisector can be used to find the **midpoint of a line segment**. A similar process can also be used to find a line **perpendicular to another line through a given point**. Follow the steps shown on the board to do each:

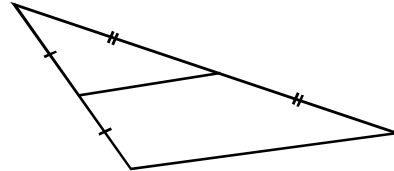
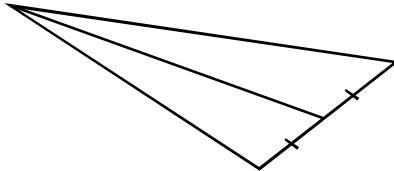
Midpoint:



Perpendicular, through point A.



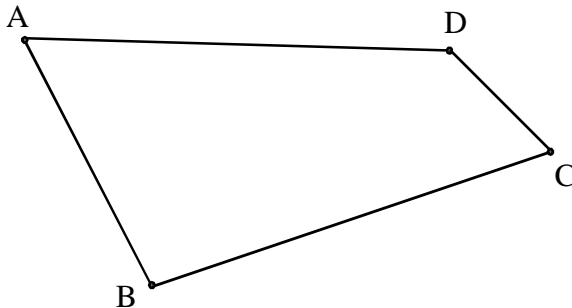
Medians and Midsegments:



Medians connect endpoint to midpoint.

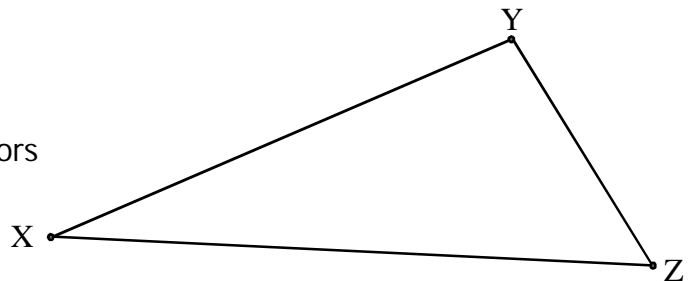
Midsegments connect midpoints.

Connect the midpoints of all sides of the quadrilateral below:



What happened?  
Will the same thing happen for every quadrilateral?  
Try a second quadrilateral on separate paper.

Draw all of the perpendicular bisectors for the triangle at the right.  
What happened?



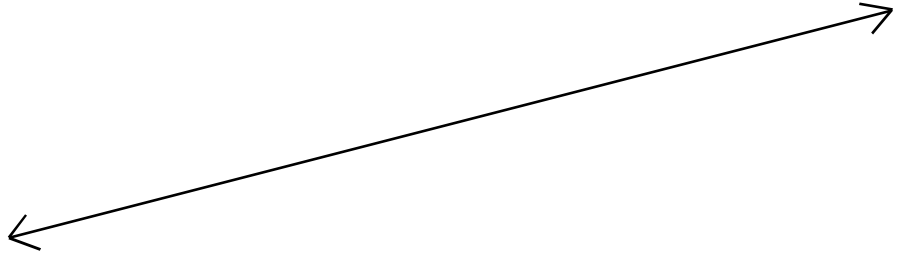
# Angle Bisecting/ Review

## Geometry 3.4

Complete each exercise below: Use **ONLY** a compass and straight-edge.  
Leave construction marks, darken or ink the final figure.

1. Construct an isosceles right triangle using the point below as one of the vertices, with one of the legs on the line below.

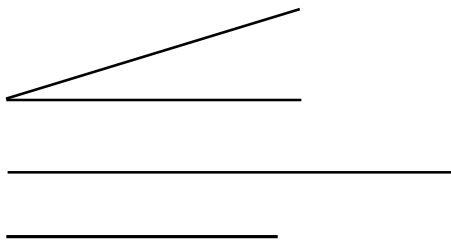
A •



2. Construct a rhombus whose sides are all equal to the segment below.  
(there are various rhombuses that will work for this problem)



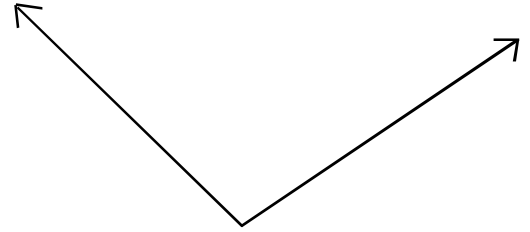
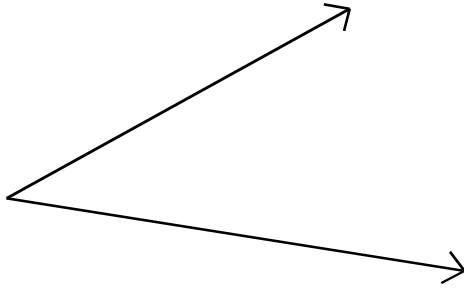
3. How many **different** triangles can you draw which contain the angle below, along with the two sides given? Use a separate sheet if necessary.



# Angle Bisecting/ Review

## Geometry 3.4

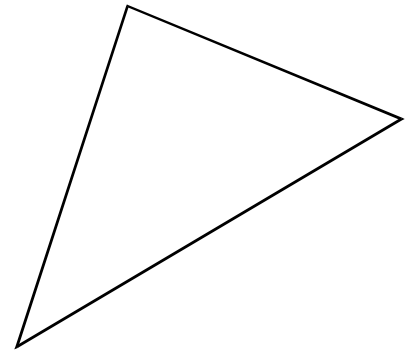
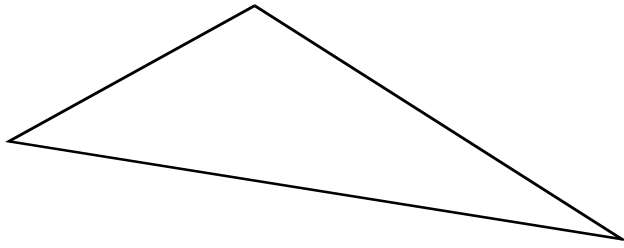
Follow the steps shown on the board to bisect each angle below:



What is the relationship between the angle's rays and its bisector?

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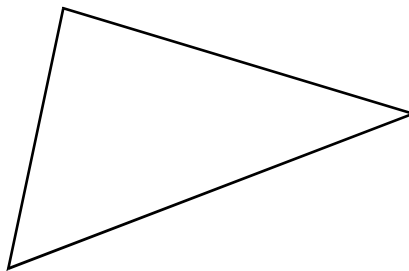
Bisect all three angles of each triangle below.



What happened? What is the significance? Why did this happen?

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Circumscribe a circle about the triangle below, and inscribe a circle within it.



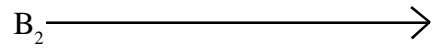
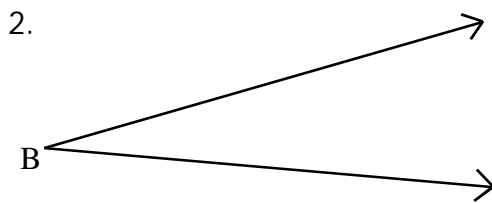
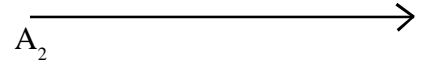
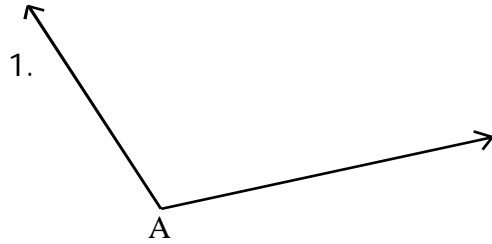
# Geometry 3.4

## Practice Quiz

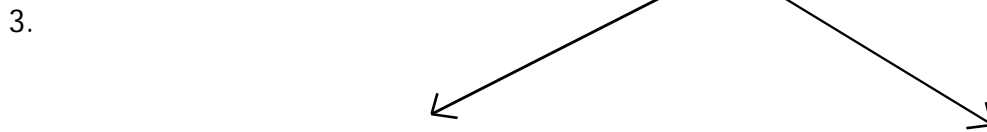
**Complete the constructions below:**

Use **ONLY** straight-edge and compass. You must have your own tools for the quiz.

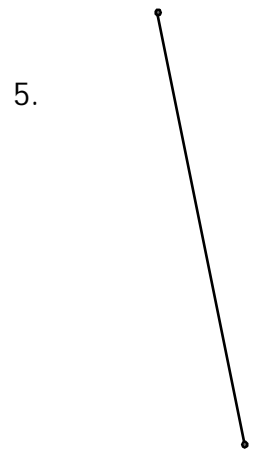
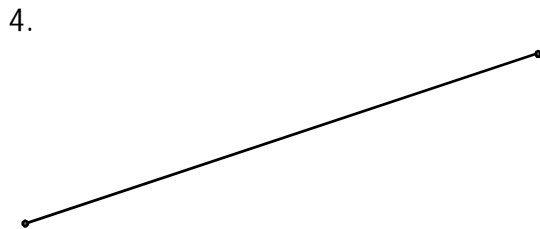
Duplicate each angle below: **SHOW ALL CONSTRUCTION MARKS.**



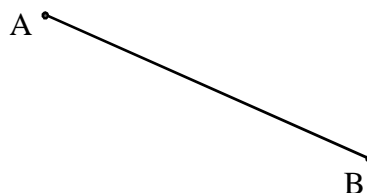
Bisect the angle below:



Construct a perpendicular bisector for each segment below:  
Leave ALL construction marks.



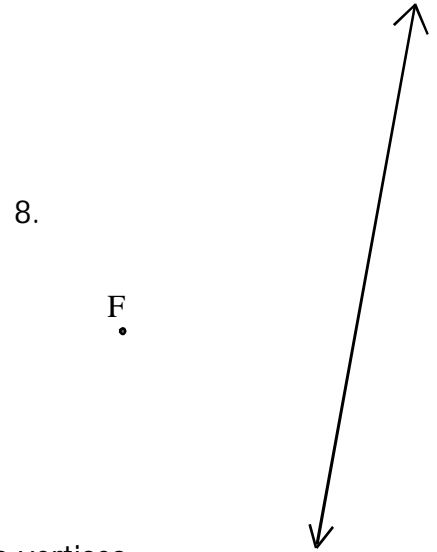
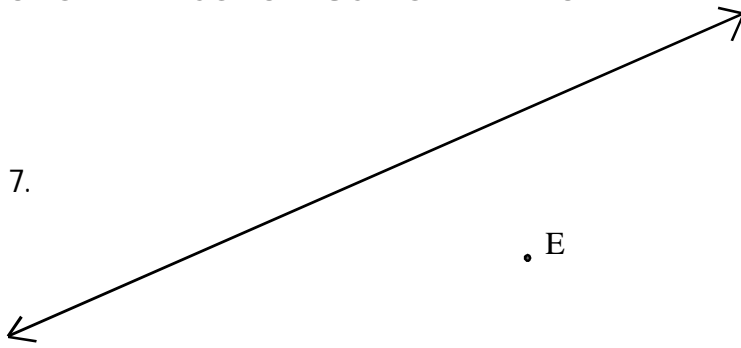
6. Construct isosceles right triangle ABC with the right angle at C.



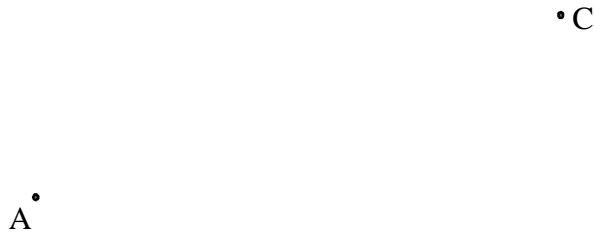
# Practice Quiz

## Geometry 3.4

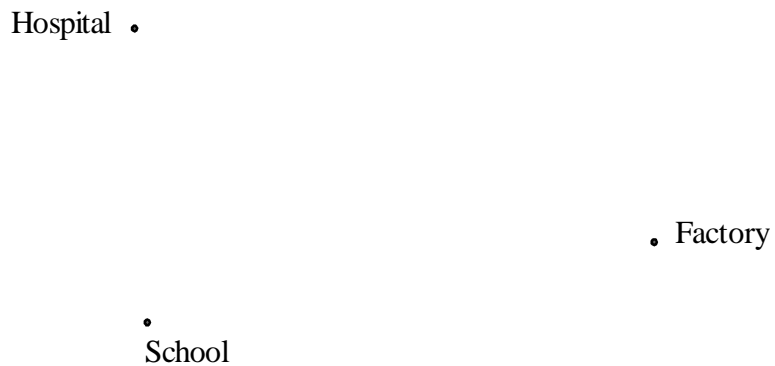
Use ONLY straight-edge and compass. You must have your own tools for the quiz. Construct a line through each point below that is perpendicular to the nearest line: **SHOW ALL CONSTRUCTION MARKS.**



9. Construct rhombus ABCD using points A and C below as vertices.



10. A fire station needs to be located so that it is exactly the same distance from each of the three locations below. Find and label the point where the station should be located (label it "fire station").



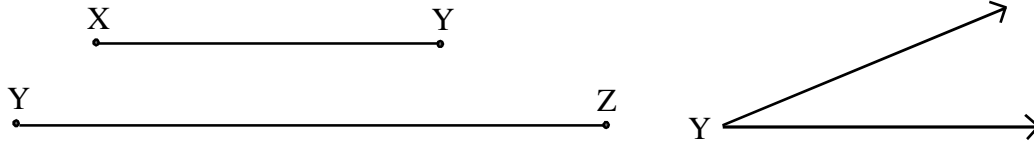




# Parallel Lines

## Geometry 3.5

Complete each construction below using the following:

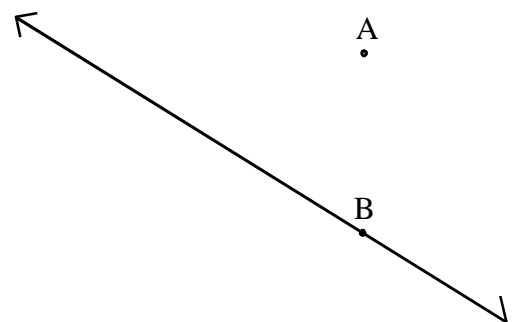


4. Construct parallel lines by creating line  $XY$ , then constructing perpendicular segments through both points ( $X$  and  $Y$ ).

5. Construct parallelogram  $WXYZ$  using the segment lengths and angle above.

6. Construct a square with segments of length  $XY$ .

7. Construct a pair of parallel lines through the points below perpendicular to the given line.



# The Centroid

For triangles, we have learned to construct the **circumcenter** (intersection of the perpendicular bisectors) and the **incenter** (intersection of angle bisectors).

The **centroid** is the intersection of a triangle's medians. Recall that the medians connect vertices to the opposite midpoint.

Construct a triangle and find its centroid.

1. Will the centroid ever be outside the perimeter of a triangle?
2. What is the significance of the medians of a triangle?
3. Can you guess the significance of the centroid?

### Activity:

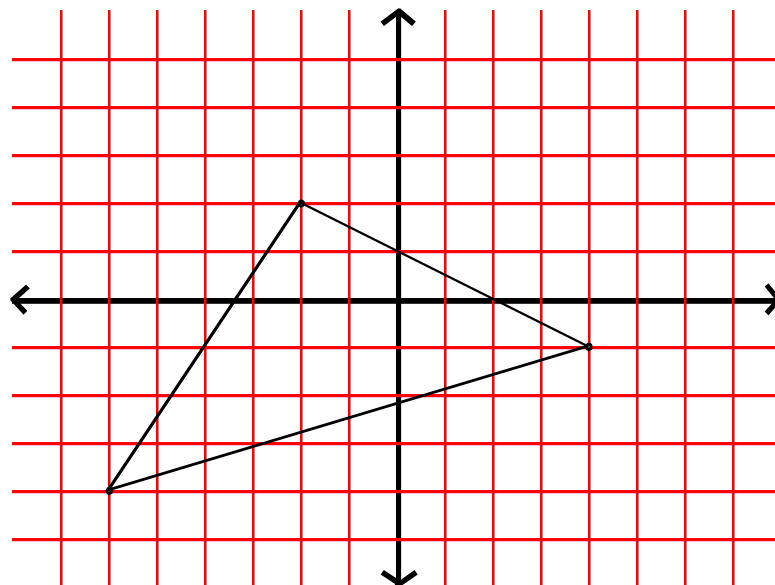
Sketch a palm-sized triangle on heavy paper and find each of the three medians. Cut the triangle out and attempt to balance the triangle along each of the three medians.

Push an indentation into the centroid using the end of your compass. Try to balance the triangle on your pencil by placing it into the indentation. Can you spin it?

### Coordinate Geometry:

We can use coordinate geometry to find a triangle's Centroid. The centroid is located by finding the mean of the x and y coordinates.

Find the centroid of the triangle described by the points below:  
Does this method seem to work?



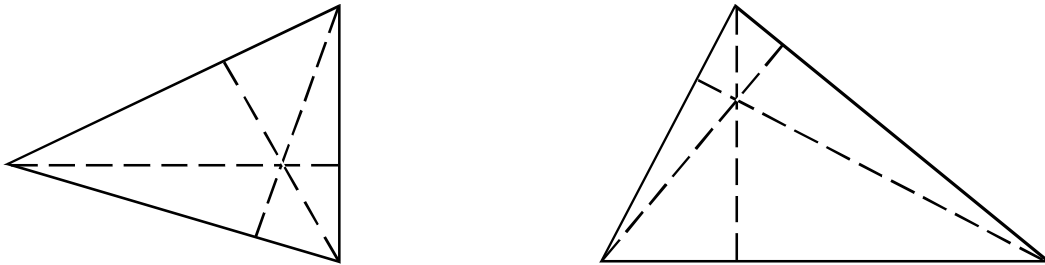
# The Orthocenter

## Geometry 3.7

The perpendicular line from a vertex of a triangle through the opposite side is called an **altitude**.

Draw an ACUTE triangle and sketch the three altitudes (you may need to extend the sides of your triangle).  
Are they concurrent?

The intersection of the three altitudes is called the **orthocenter**.



### Questions:

Where will the orthocenter of a right triangle be located?

Where will the orthocenter of an obtuse triangle be located?

### The usefulness of the orthocenter... is virtually nonexistent.

However, it has an interesting relationship to two of the three other points we have learned to construct.

**Construct a triangle.** Any triangle will work, if the person next to you is making an acute triangle, make your obtuse. Try to make the longest side as large as your compass will open. **Trace it in ink.**

Find the **orthocenter, circumcenter, incenter, and centroid**. You will probably need to erase construction lines along the way.

Accuracy is important.

Which three points are collinear?

The Centroid, Circumcenter, and Orthocenter are always collinear and form a line called the Euler Line (which, like the orthocenter, is essentially useless in any practical applications).

### The Euler Circle (nine-point circle)

Construct a circle through the midpoints of the sides of any triangle ABC.

Construct the altitudes of the same triangle ABC.

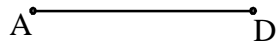
What did you notice?

# Constructions Practice Quiz

## Geometry 3.4

Use ONLY straight-edge and compass.

1. Construct isosceles triangle ABC with base BC and altitude length AD (given).



•  
B

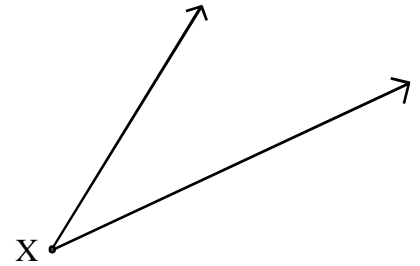
•  
C

3. Construct square GHIJ.

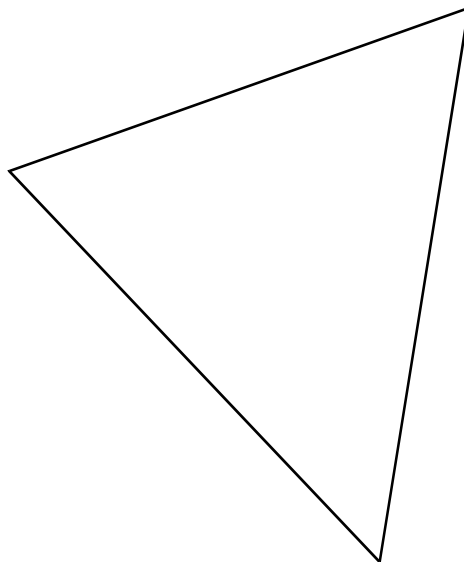
•  
G

•  
H

2. XY is the angle bisector of angle WXZ. Draw ray XZ.



4. Circumscribe a circle about the triangle below, then inscribe a circle within it. Leave all construction marks.



# Geometry 3.8

## Constructions Practice Quiz

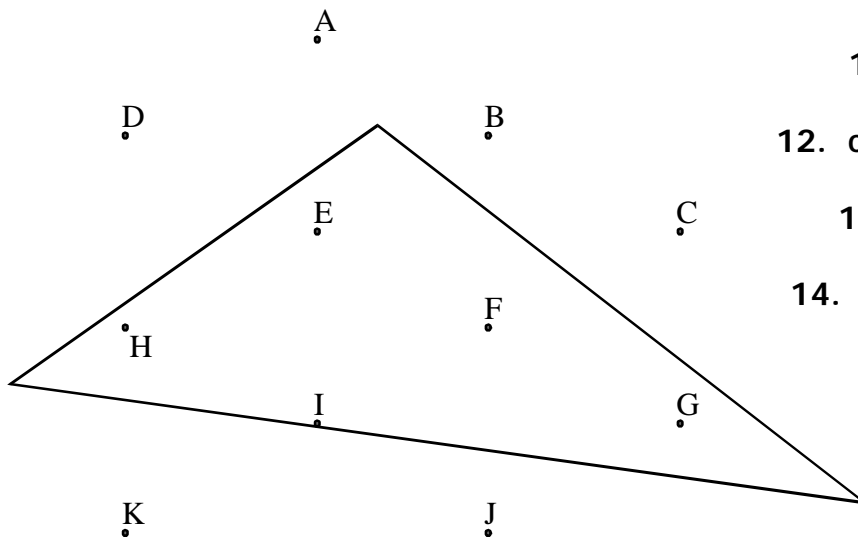
Complete each of the following statements:

- |  |           |
|--|-----------|
| 5. The centroid is the intersection of a triangle's _____.         | 5. _____  |
| 6. The circumcenter is the intersection of a triangle's _____.     | 6. _____  |
| 7. The incenter is the intersection of a triangle's _____.         | 7. _____  |
| 8. The orthocenter is the intersection of a triangle's _____.      | 8. _____  |
| 9. The triangle center that is not on the Euler line is the _____. | 9. _____  |
| 10. A triangle's center of gravity is its _____.                   | 10. _____ |

**On the triangle below, locate the following and label each:**

- in:** The Incenter
  - cc:** The Circumcenter
  - cd:** The Centroid
  - oc:** The Orthocenter
- You may erase as you go.

Find the point **NEAREST** each of the four centers.



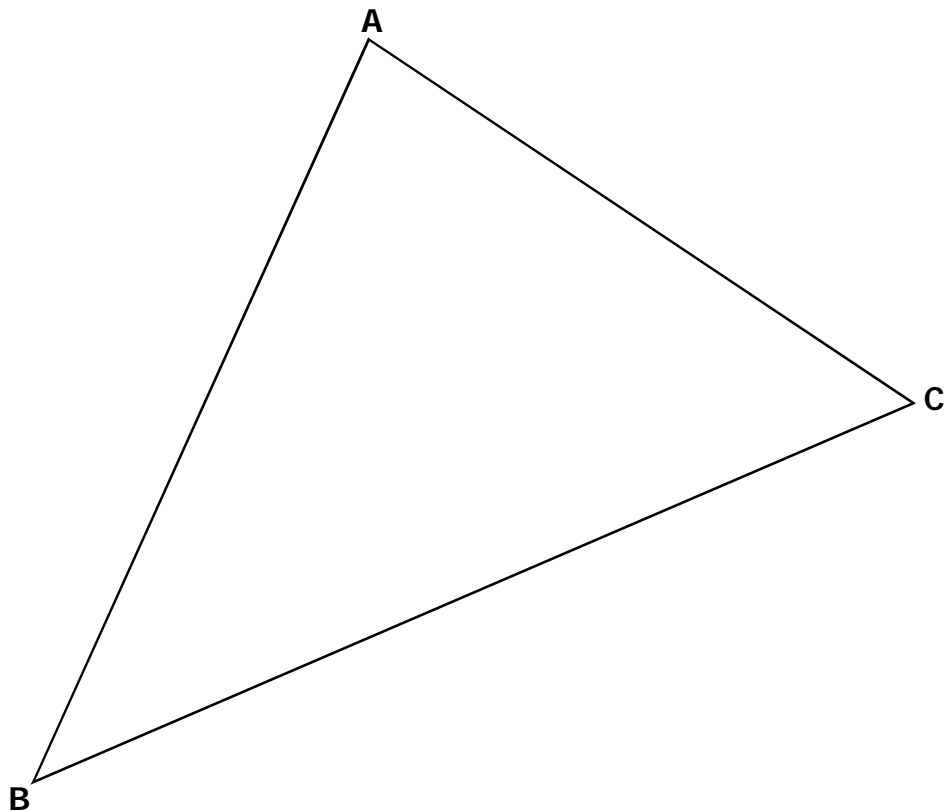
- 11. in:** The Incenter \_\_\_\_\_
- 12. cc:** The Circumcenter \_\_\_\_\_
- 13. cd:** The Centroid \_\_\_\_\_
- 14. oc:** The Orthocenter \_\_\_\_\_

You may use the same letter twice.

# Nine-point circle construction:

## Geometry 3.8

1. Find the midpoints of the sides of triangle ABC below.
  2. Construct a circle which passes through these midpoints (you will need to find the circumcenter of the triangle which has these three points as vertices).
  3. Construct the altitudes of triangle ABC (do you notice anything about their relation to the circle?)
  4. Mark the orthocenter point T.
  5. Draw AT, BT, and CT.
  6. Find the midpoints of AT, BT, and CT. Do they have any relationship to the circle?
  7. Describe the nine points that are part of the Euler (nine-point) circle:
- 
- 







# More Difficult Constructions

## Geometry 3.6

**Complete each construction below:**

4. You know how to bisect angles, and you know how to draw perpendicular lines. Use these skills to create a regular octagon.

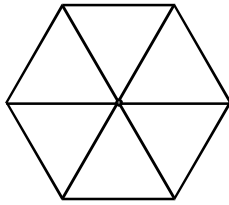
Hint: Start at the center of the octagon.

5. Create isosceles trapezoid  $WXYZ$  in which the short base ( $XY$ ) and the two sides ( $WX$  and  $YZ$ ) are all equal in length, and the long base ( $WZ$ ) is twice the length of the short base.

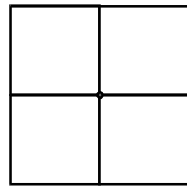


# Tessellations:

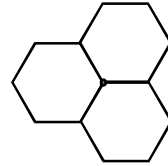
There are three regular tessellations (below) composed of regular polygons which tile a plane:



3.3.3.3.3.3



4.4.4.4

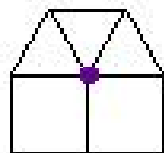


6.6.6

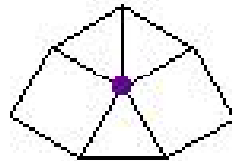
The tessellation naming convention involves naming the number of sides of each polygon surrounding every vertex.

**Choose one of the eight semi-regular tessellations below and construct the semi-regular tessellation that it defines.**

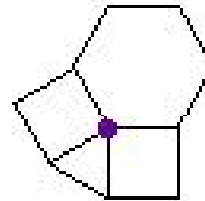
Start with a sketch of how the plane will be tessellated, then figure out how you can construct the tessellation using only a compass and straight-edge.



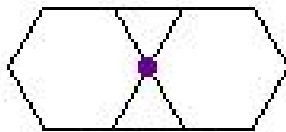
**3.3.3.4.4**



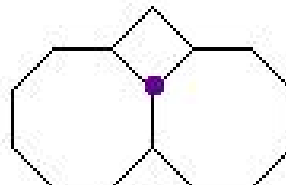
**3.3.4.3.4**



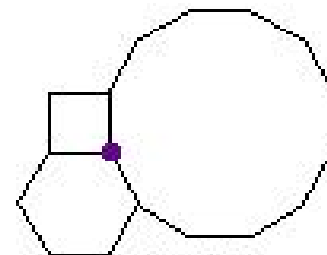
**3.4.6.4**



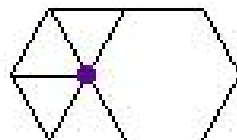
**3.6.3.6**



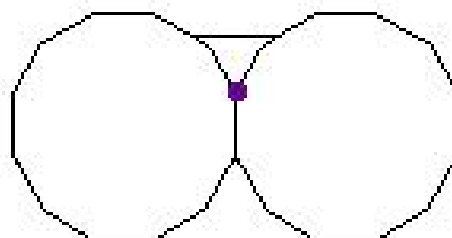
**4.8.8**



**4.6.12**



**3.3.3.3.6**



**3.12.12**

# Circumradius, Inradius.

There are many formulas related to the circumradius and inradius of a right triangle.

### Easy:

Construct a right triangle. Label the sides  $a$ ,  $b$ , and  $c$  (where  $c$  is the hypotenuse).

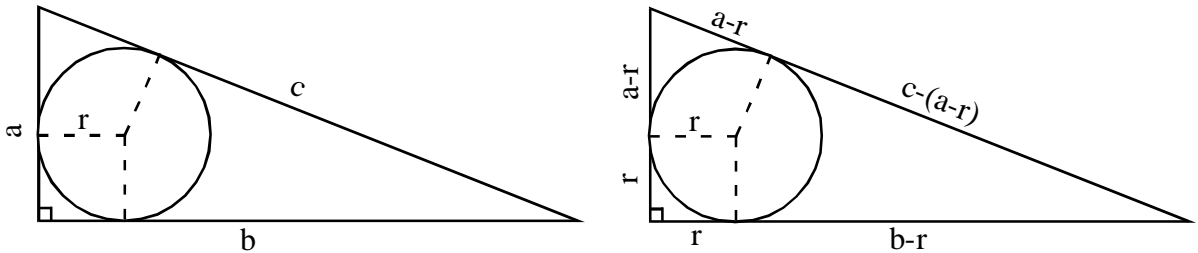
Find the circumcenter. The circumradius is the *radius* of the circumscribed circle. The circumradius is usually labeled with a capital  $R$ . Write a formula for the circumradius  $R$  of a right triangle.

### Medium:

Construct a right triangle. Label the sides  $a$ ,  $b$ , and  $c$  (where  $c$  is the hypotenuse).

Find the incenter. The inradius is the radius of the inscribed circle. The inradius is usually labeled with a lower-case  $r$ . Write a formula for the inradius of a right triangle based on  $a$ ,  $b$ , and  $c$ .

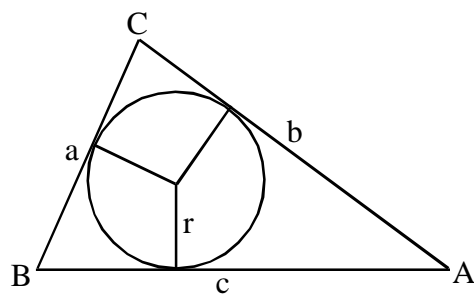
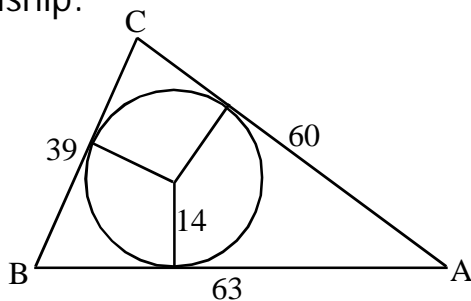
hint:



### Hard:

For any triangle (not just right triangles) there is a relationship between the inradius  $r$ , the area  $A$ , and the perimeter  $P$ .

Find the area of the triangle below, and you will likely be able to find this relationship:



The area formula:  $A = rP/2$

# Inradius/Circumradius

## Geometry

Use the formulas we discovered in class to solve each:

$$R = \frac{1}{2}c \quad r = \frac{a+b-c}{2} \quad A = \frac{rP}{2}$$

(Learn where these come from and you don't ever need to memorize them!)

*R: circumradius r: inradius A: triangle area P: triangle perimeter*

1. What are the inradius and circumradius of an 8-15-17 right triangle?  
(8-15-17 means the side lengths are:  $a=8$ ,  $b=15$ ,  $c=17$ )
2. What is the approximate area of a triangle whose sides measure 11, 15, and 20cm and whose inradius is about 3.5cm?
3. The perimeter of a triangle whose area is  $67\text{cm}^2$  is 42cm. What is the radius of the inscribed circle?
4. The area of a triangle with sides measuring 30, 50, and 60cm is about  $748\text{cm}^2$ . Approximate the inradius of the triangle to the nearest tenth of a centimeter.
5. What is the area of the circle inscribed within a 9-40-41 right triangle?  
(circle area =  $\pi r^2$ )

# Inradius/Circumradius

# Geometry

Use the formulas we discovered in class to solve each:

6. A triangle with sides measuring 10, 10, and 14cm is almost a right triangle. Approximate the inradius and circumradius of a 10-10-14 triangle.

7. The actual hypotenuse of an isosceles right triangle whose legs are 10cm is  $10\sqrt{2}$  cm. What is the area of the circumscribed circle in terms of Pi?

8. What formula could be used to find the inradius ( $r$ ) of a triangle given the area ( $A$ ) and the semi-perimeter ( $s$ )? The semi-perimeter is equal to half of the perimeter and is used in many formulas involving triangles. Write your formula for  $r$  in terms of  $A$  and  $s$ .

9. The area of a triangle is  $300\text{cm}^2$ . Two of the sides measure 24 and 30 cm. If the inradius is 7.5cm, what is the length of the third side?

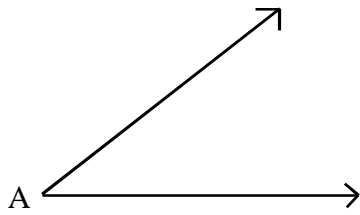
**challenge:** A 6-8-10 triangle has an inscribed circle and a circumscribed circle. What is the distance from the incenter to the circumcenter? Leave your answer in simplest radical form.

# Practice Test

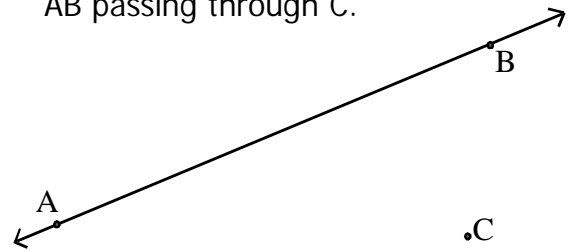
## Geometry 3.8

**Complete each construction below:** Leave ALL construction marks.

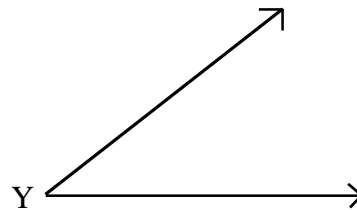
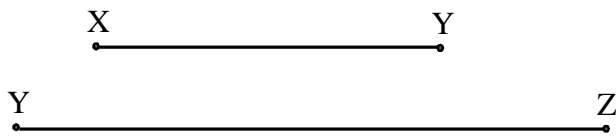
1. Bisect the angle below:



2. Create a line perpendicular to AB passing through C.



Use the following for #3-5:



3. Construct rectangle WXYZ.

4. Construct parallelogram WXYZ using the segment lengths and angle above.

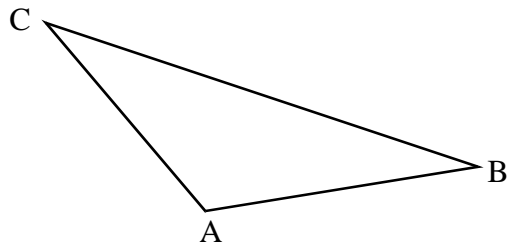
5. Construct a square with segments of length XY.

# Practice Test

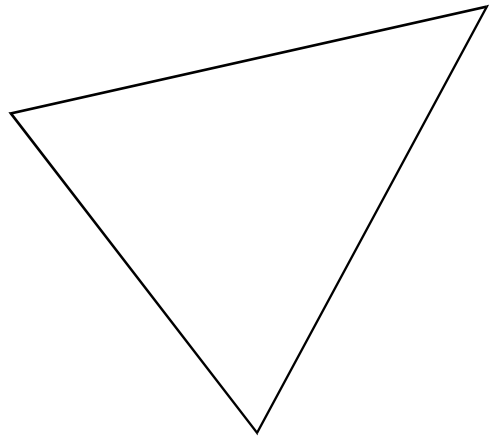
## Geometry 3.8

Complete each construction below. Leave all construction marks.

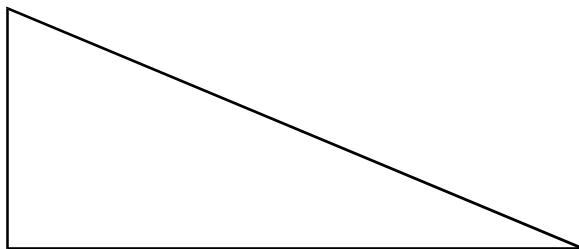
6. Construct altitude CD:



7. Inscribe a circle within triangle DEF.

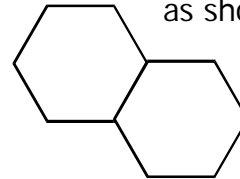


8. Find the center of gravity for the figure below:



9. Sketch neatly (do not construct, but please use a straight edge) the five shapes surrounding a single vertex of a 3.3.4.3.4 tessellation.

10. Construct adjoining hexagons as shown (larger please):



Use separate paper if needed.

11. What is the inradius of a 20-21-29 right triangle?

11. \_\_\_\_\_

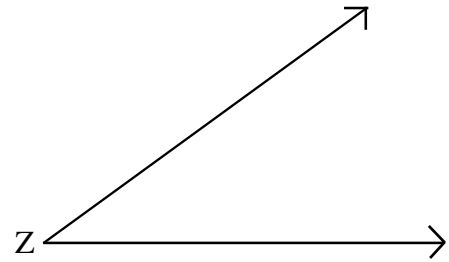
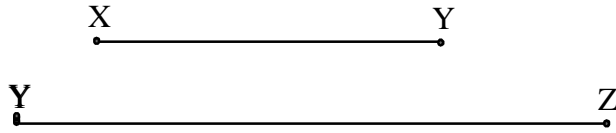
# Test Review

## Geometry 3.8

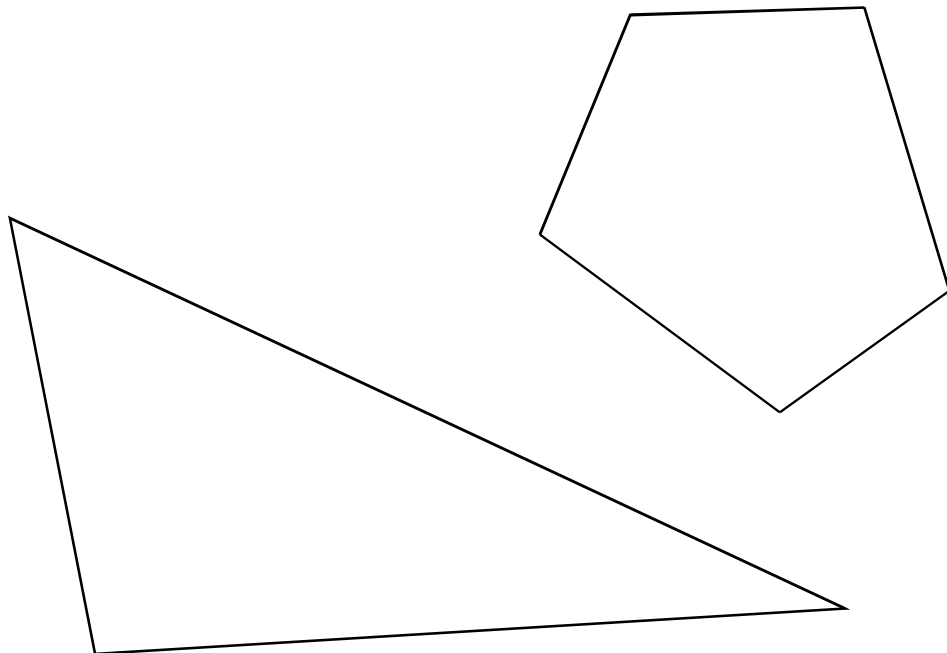
Your team should attempt to complete each of the following tasks for the point values given:

Use the following wherever applicable.

Use separate paper for work.



- 100 Duplicate YZ and find its midpoint.
  - 100 Construct equilateral triangle WXY.
  - 100 Construct angle XYZ.
  - 200 Duplicate XY and bisect it with a perpendicular.
  - 200 Construct right triangle XYZ.
  - 200 Construct Isosceles triangle WYZ.
  - 300 Construct Kite WXYZ.
  - 300 Construct a hexagon with sides of length XY.
  - 300 Construct square VWXY.
  - 400 Construct parallelogram WXYZ with two angles measuring  $135^\circ$ .
  - 400 Construct a regular star using the angle above.
  - 400 Construct three congruent regular hexagons which share a common vertex.
  - 500 Construct a regular octagon with sides of length XY.
  - 500 Construct a regular dodecagon (12 sides)
  - 500 Construct the Euler line for the triangle below.
  - 500 Find (construct, no guessing) the circumcenter of the pentagon below and draw the circumscribed circle (note: not all pentagons can be inscribed within a circle).
- total: out of 5,000pts \_\_\_\_\_.**





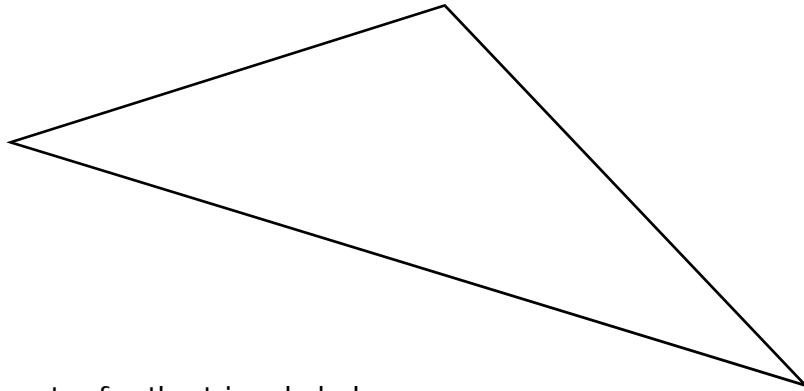
# Review: Concurrency

# Geometry Re

Complete each construction below:

Find the incenter for the triangle below. The incenter is the center of the inscribed circle within the triangle.

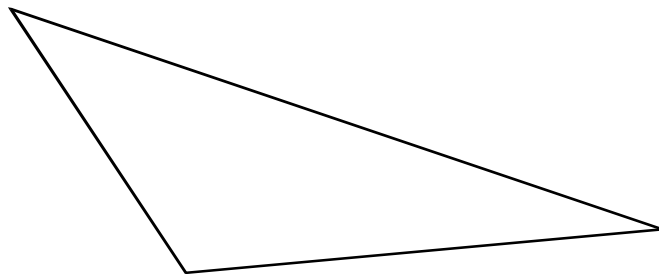
Use the \_\_\_\_\_.



Find the circumcenter for the triangle below.

The circumcenter is the center of a circumscribed circle about the triangle.

Use the \_\_\_\_\_.



A cell phone company is placing a new tower. The company has decided that the new tower must be placed exactly the same distance from each of the three existing towers it has already built in the area. Mark the best location for this tower on the map below.

Tower A •

• Tower C

•  
Tower B

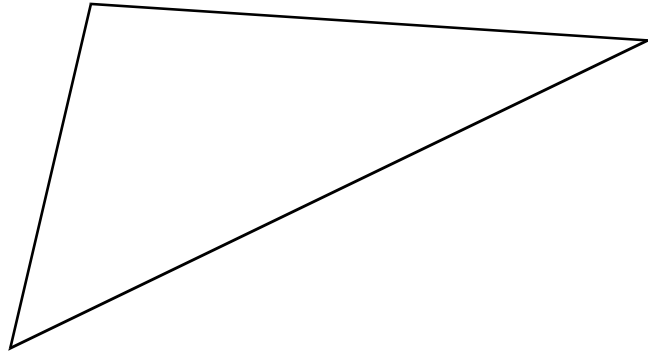
# Review: Concurrency

# Geometry Re

Complete each construction below:

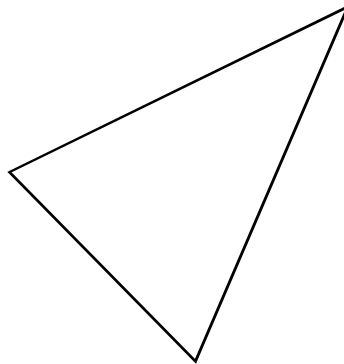
Find the centroid of the triangle below. The centroid is the center of gravity of the triangle.

Use the \_\_\_\_\_.

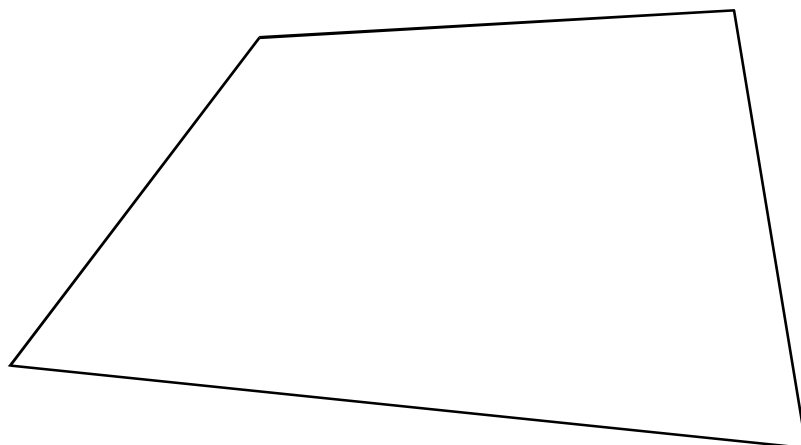


Find the orthocenter of the triangle below. The orthocenter has almost no practical use, but it occurs on the Euler Line along with the circumcenter and the centroid.

Use the \_\_\_\_\_.



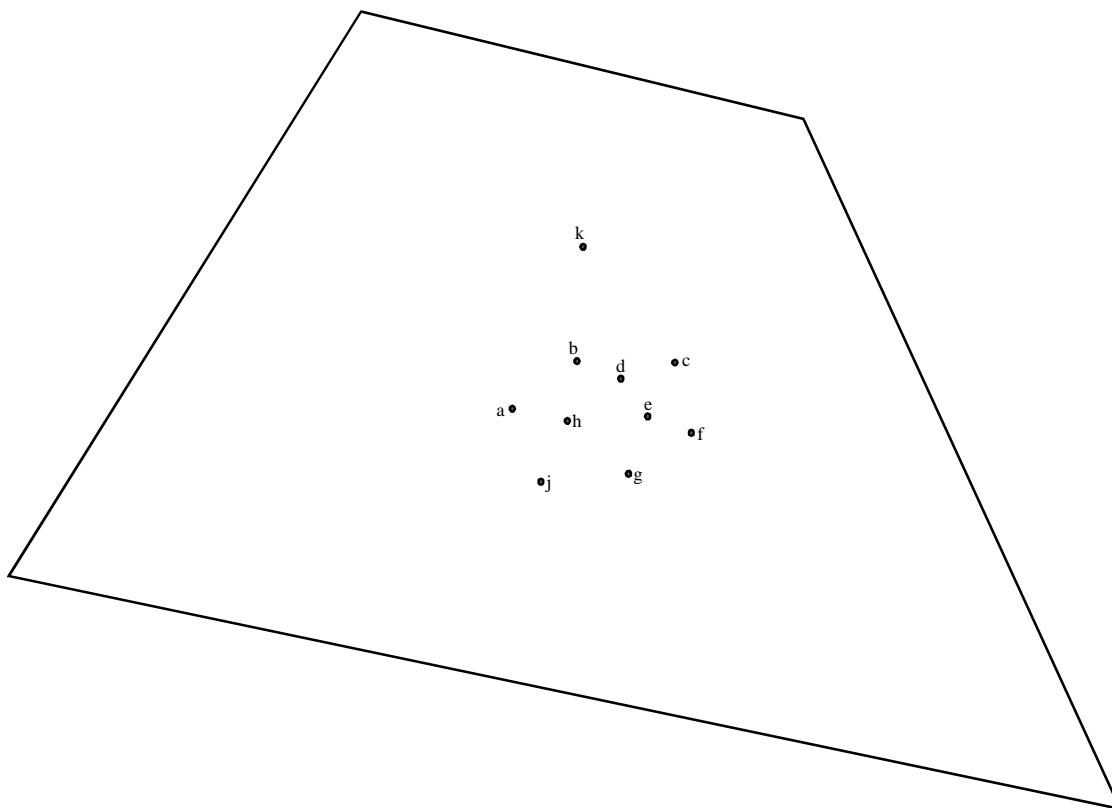
Think! Draw a line that divides the quadrilateral below into two halves of equal area.



# Review: Concurrency

# Geometry Re

Find the center of gravity of the quadrilateral below.



# Review: Concurrency

# Geometry Re

**Find the point that is the circumcenter of the of the quadrilateral below.**  
Do not guess-and-check.

Draw the circle that circumscribes the quadrilateral.

