# Ten Easy Tricks and Skills

Everyone should know.

1. ___________ $cm^2$ In triangle ABC, angle A = 60 degrees and AB = 4cm, and AC = 10cm. What is the area of triangle ABC? Express your answer in simplest radical form.

2. ____________ What is the probability of rolling a 5 with a pair of standard six-faced dice?

3. ____________ How many different ways can the letters in the word ARRANGE be arranged?

4. ____________ How many positive integers are factors of 540?

5. ____________ What is the units digit of $1{,}234^{1{,}234}$?

6. ____________ How many four-digit integers use each of the digits 1, 2, 3, and 4 exactly once and are divisible by 11?

7. ____________ In the senior class at Hillside High School there are 200 students. 156 students will attend a university next fall, 67 students will be employed, and 35 students will be employed while attending a university. How many students will not work or attend a university?

8. ____________ What is the sum of the 40 smallest positive multiples of 7?

9. ____________ In the prime factorization of 100!, what is the power of 5?

10. ____________ What is the greatest prime factor of $2^{10} - 1$?

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**Solutions:**

1. Drop the altitude from B to AC with the foot of the altitude at D. This creates 30-60-90 triangle ABD. AB=4 so AD=2 and BD (the altitude of triangle ABC) = $2\sqrt{3}$cm, making the area of triangle ABC: $\frac{10 \cdot 2\sqrt{3}}{2}$ = $\mathbf{10\sqrt{3}.}$

Q1: What is the area in square units of an equilateral triangle with a perimeter of 8 units? Express your answer as a common fraction in simplest radical form.

2. There are $6^2$ possible outcomes for two rolls of a pair of dice. There is one way to roll a 2 (1,1), there are 2 ways to roll a 3, three ways to roll a 4, four ways to roll a 5, five ways o roll a 6, six ways to roll a 7, 5 to roll an 8, 4 to roll a 9, 3 to roll a 10, 2 to roll an 11, and 1 way to roll a 12. We are asked for P(5), so 4/36 = **1/9**.

Q2: What is the probability of rolling an 11 with a standard pair of dice?

3. There are 7 letters. If they were all different, we could choose 7 to to first, 6 second, 5 thirds … etc., so 7! = 5,040. However, there are two A's and two R's in ARRANGE. Number them to see that $A_1R_1R_2A_2$NGE is the same as $A_2R_1R_2A_1$NGE and $A_1R_2R_1A_2$NGE and $A_2R_2R_1A_1$NGE because there are (2!) ways to arrange the A's and (2!) ways to arrange the R's. We must divide by (2!)(2!) to get **1,260** arrangements.

Q3: How many ways can six students stand in line from left to right if two of the students, Lisa and John, are next to each other?

4. 540 = $2^2$x$3^3$x5. Each factor is a combination of 2's, 3's, and 5's … 0, 1, or 2 twos, 0, 1, 2, or 3 threes, and either 0 or 1 five. This gives us 3 choices for the number of 2's times 4 choices for the number of 3's, and two choices for the number of 5's. 3 x 4 x 2 = **24** factors.

Q4: What number could fill-in the blank in 52,8_7 to make the number divisible by 11?

5. We only care about the units digit. $4^2$ = 4x4 which ends in a 6. Multiply by 4 again and we get a number that ends in a 4. Multiply by 4 again and we get a number that ends in a 6, and so forth: $4^1$ ends in a 4, $4^2$ ends in a 6, $4^3$ ends in a 4, etc. Every even power ends in a 6, so 1,234 ends in a **6**. Every units digit has a pattern that makes problems like these easy.

Q5: What is the units digit of $123^{123}$?

6. The divisibility rule for 11: Add alternating digits. Subtract these two sums. If the result is divisible by 11, the integer is a multiple of 11. For example, with 45,870 we have 3**5**,8**7**1 (5+7=12) and **3**5,**8**7**1** (3+8+1=12), so 12-12=0 means that 35,871 is divisible by 11. To get equal alternating digit sums using 1, 2, 3, and 4 we use 1&4 and 2&3 as alternating digits. We can begin with any one of the four digits, and there are two choices for the second digit. After that, the last two digits are fixed. We have:
1,243 1,342 2,134 2,431 3,124 3421 4,213 and 4,312. There are therefore **8** four-digit integers which use 1, 2, 3, and 4 and are divisible by 11.

7. Use a Venn diagram or the following reasoning: There are 156 university students and 67 workers, but the 35 who do both all got counted twice, so there are 156 + 67 – 35 = 188 students doing one, the other, or both. This leaves **12** to do neither.

Q7: There are 35 dogs at the pound. 17 are brown, 12 have spots, and 8 are not brown and don't have spots. How many of the dogs at the pound are brown and spotted?

8. This is the same as 7(1+2+…+39+40) which is $7 \cdot \frac{40(41)}{2}$ = 7 x 20 x 41 = **5,740**. The sum of 1+2+…+n is $\frac{n(n+1)}{2}$.

Q8: Find the sum: 11+17+…+305.

9. In the product 100x99x98x97…x5x4x3x2x1 there are many multiples of 5 which contribute a power of 5 to its prime factorization. Every multiple of 5 contributes one 5. Every multiple of 25 contributes two 5's. There are 100/5=20 multiples of 5, plus another 100/25 = 4 multiples of 25 (each has been counted once already, so we just add four more). 20 + 4 = **24**, so $5^{24}$ is the power of 5 in the prime factorization of 100!.

Q9: When written out as a long whole number, how many zeroes does 125! end in?

10. This requires a little Algebra that you may not have seen called factoring a difference of squares. Basically, $a^2$-$b^2$ = (a+b)(a-b), so $2^{10}$-1 = ($2^5$+1)($2^5$-1) = 33x31, so **31** is the greatest prime factor of $2^{10}$-1.

Q10: How many factors does the integer represented by $81^2$-$49^2$ have?

Want to know one or more answers to the follow-up question? Feel free to write me at jbatterson@agmath.com.