Solutions: Challenge 5

1. As you eat, there are either 5 or 4 more red candies than orange … 19/14, 18/14, 18/13 … we eat until there are 6 red candies remaining and 2 orange candies. We start with 33 and end with 8, so we have eaten **25**.

2. Note that 5/15 is equal to 1/3 and 5/7.5 is equal to two-thirds. There are **7** integer values between 7.5 and 15.

3. One $\frac{1}{2}$-ounce hummingbird would eat an ounce of nectar a day, so in 7 days, 8 hummingbirds would eat 8x7=**56** ounces of nectar.

4. If $\frac{1}{x+1} = 7$, then $x+1 = \frac{1}{7}$, so $x = -\frac{6}{7}$. If $\frac{1}{y-1} = 7$, then $y-1 = \frac{1}{7}$, so we have $y = \frac{8}{7}$. Therefore, $x+y = \frac{2}{7}$ and $\frac{1}{x+y} =$ 7/2 or $3\frac{1}{2}$.

5. A number that is divisible by 99 is divisible by both 9 and 11. It is impossible to write a 4-digit palindrome that is not divisible by 11 (alternating digit sums will always be equal). To make the number divisible by 9, the digit sum must be divisible by 9. Think of this as placing two 2-digit numbers, one frontwards and one backwards. Each 2-digit number must be divisible by 9 (ie 4554). We can use any 2-digit multiple of 9 starting with 18 and ending with 99: 1881, 2772, … 9999. This gives us **10** palindromes.

6. Any arrangement can be rotated so that Pamela appears to be in the 12-o'clock position (on a clock, label the 6 positions 12, 2, 4, 6, 8, and 10-o'clock. This saves me the trouble of drawing a diagram). Now, place Corey … there are 3 places where Corey can be in the arrangement (4, 6, or 8-o'clock). The four remaining people can then be placed in 4! = 24 ways. 3x24=**72** possible arrangements.

7. Think of this as hopping up the number line … we hop 36 spaces from 101 to 137 and 72 spaces from 137 to 209. The number of spaces we hop must be a factor of both 36 and 72. This obviously occurs and time a number is a factor of 36. There are **9** factors of 36 (1, 2, 3, 4, 6, 9, 12, 18, and 36), and any arithmetic sequence that passes through 101 and has a difference of one of these factors will also pass through 137 and 209.

8. Let's imagine this cube on the 3-dimensional coordinate axis with its faces parallel to the axis and its center at (0,0). The sphere intersects the edges of the cube so that the edges are trisected into 2cm segments. Now you will need to use your imagination … this diagram is too hard for me to draw with the limited tools I have … the points of intersection are 3cm from the center along one axis, 3cm away along another axis, and 1cm from the center on the 3rd axis. The distance formula in 3-dimensions is $d=\sqrt{x^2+y^2+z^2}$ so we have $\sqrt{19}$ as the distance from the center of the cube to a point of intersection, which is the radius of the sphere. The surface area of a sphere is given by $4\pi r^2$, so we have a surface area of $\mathbf{76\pi}$ cm^2.

9. This is another sticks-and-stones problem (see #3 in set 2) but with added complications.
There are 21 – 3 = 18 goals to distribute to the three players, so 2 sticks and 18 stones gives us ${}_{20}C_2$ = 190 ways to distribute the goals without restrictions, however, the problem states that no

two players scored the same number of goals. This eliminates possibilities like 0-0-18 (3 times, because 1-16-1 and 16-1-1 are different distributions). These distributions:

0-0-18

1-1-16

2-2-14,

3-3-12,

4-4-10,

…

9-9-0 all occur three ways except 6-6-6. 9(3) + 1 = 28 distributions which must be eliminated.

190 - 28 = **162**.

10. The greatest distance between them is the hypotenuse of a right triangle whose sides are 13 meters long. $\sqrt{338}$ is a little more than 18. The smallest distance between them occurs when they are 13m apart. At some point very early on in the walk they will be 18m apart, then 17m, then 16 … 15 … 14 … 13 … then back up, 14, 15, 16, 17, and 18m apart. This is **11** times.

Thanks for all who participated! I wrote these pretty fast …

-Mr. B.