Area

Rectangles, Parallelograms, Triangles, Trapezoids, Kites:

1. What formula can you use for the area of any parallelogram (including rectangles, squares, and rhombuses)? Diagram why this formula will work even for the ‘tilted’ shapes.

2. What formula is used to determine the area of a triangle? Explain why it always works.

3. What is the formula for the area of a trapezoid? Can you create a diagram which demonstrates why this formula works?

4. What is the formula for the area of a kite? Can you explain why this formula works?
Area

Find the area of each:

Rectangle $ACDG$ has an area of $90\text{cm}^2$.

1. What is the area of triangle $BDG$?
2. What is the area of triangle $BFD$?
3. $AB=4$, $AG=6$. Find the area of trapezoid $BCDF$.

$ABCD$ is a kite (not drawn to scale). $AC=50$, $BD=48$, $AB=30$.

1. What is the area of kite $ABCD$?
2. What is the length of $BC$?
3. Find the area of triangle $ABX$.

Triangle $VRT$ has an area of $180\text{cm}^2$ (not to scale).

1. What is the area of triangle $RWT$?
2. What is the area of triangle $RST$?
3. What is the area of triangle $VTS$?
**Area Practice**

Find the area of each figure listed:

1. Triangle SAR: ______
2. Trapez. MASE:______
3. Triangle PEB: ______
4. Triangle BPL: ______
5. Triangle PML: ______
6. Pent. BRALP: ______
7. Trap. BEML: ______

Answer: Explain each on the back of this sheet.

8. Is the area of all the interior figures equal to the area of rectangle RALB?

9. Is BLMP a kite?

10. Determine the length of segment PL.

11. Can you determine the length of segment MY?

**Find the area of each lettered piece:**

12. What is the area of the octagon?

13. Label the length of the missing sides of the octagon.
Mass Points

Mass Points Geometry
I have been looking for a good place to include this lesson and this seems like as good a place as any ... it was developed by HIGH SCHOOL STUDENTS in the 60’s for use in math competitions as a ‘trick’ to simplify complex problems.

Consider the following see-saw problem:
What mass of the box on the right is required to balance the box on the left?

Consider a similar problem:
Think of the see-saw as a big metal triangle of uniform thickness. The same principal holds true. If we place a weight on the left and a weight on the right, mass times distance must be equal on both sides. Try a few examples.

Practice: Find the mass in grams \( x \) at each vertex.
Mass Points

Now for some complexity ... but we will stick to the simplest cases where mass point geometry is useful.

Example. Try the following:
Find AX:DX and EX:BX.

Practice:
1. AB:BC = 1:4
   ED:DC = 1:2
   Find EX:BX and AX:DX

2. (harder)
   WX:YX = 2:3
   WO:ZO = 6:5
   Find XO:VO and YZ:VZ

So, what does this have to do with area?

The area of triangle ABC is 36cm².
If BC=2AB and CD:DE = 3:5, find the area of each polygon listed:

1. Triangle ABE: _______
2. Triangle ADE: _______
3. Triangle AXE: _______
4. Triangle DXE: _______
5. Triangle ABX: _______
6. Quadrilateral BCDX: _______
Area of Regular Polygons

How could you determine the area of the triangle below?

![Triangle Diagram](triangle.png)

Use similar strategy to determine the area of the hexagon below:

![Hexagon Diagram](hexagon.png)

The two special cases above are easier because the height of the triangle can be determined using the Pythagorean Theorem. For most polygons, this is not possible without using trigonometry.

**What formula** could be used to determine the area of a regular polygon given the:
- Number of sides: \( n \)
- Side length: \( s \)
- Apothem (inradius): \( a \)

How can this formula be simplified given the perimeter \( P \) of the polygon?

**Find the area of each regular polygon below:**

1. A nonagon whose side length is 12 cm and whose apothem is 16.5 cm?

2. A polygon whose perimeter is 60 inches and whose apothem is 8.5 in?
Area of Regular Polygons

Determine the area of each figure below:

1. \[ \text{Area} = 585 \text{ m}^2 \]

\[ \text{A} \rightarrow \text{p} \text{ in} \] \[ \text{P} \rightarrow \text{t} \text{ in} \] 

2. \[ \text{Area} = 364 \text{ in}^2 \]

What is the perimeter of each figure below? (round to the tenth)

1. \[ \text{Area} = 121 \text{ cm}^2 \]

\[ \text{A} \rightarrow \text{p} \text{ cm} \] \[ \text{P} \rightarrow \text{t} \text{ cm} \] 

2. \[ \text{Area} = 1075 \text{ ft}^2 \]

What is the apothem of each regular polygon below? (round to the tenth)

1. \[ \text{Area} = 121 \text{ cm}^2 \]

\[ \text{A} \rightarrow \text{p} \text{ cm} \] \[ \text{P} \rightarrow \text{t} \text{ cm} \] 

2. \[ \text{Area} = 1075 \text{ ft}^2 \]
**Area of Regular Polygons**

Determine the area of each shaded figure below:

1. \(a = 4.4\ \text{cm}\)
   \(s = 6\ \text{cm}\)

2. Round to the tenth.

**What is the outside perimeter of each figure below? (round to the tenth)**

1. Shaded Area = 170\(m^2\)
   \(a = 8.5\)

2. Shaded Area = 91 \(\text{in}^2\)

**Find the area of the shaded region below:**
Round to the tenth.
**Area of Circles**

**Complete the activity** below to explain why the area of a circle is equal to pi times the radius squared:

Sketch a circle sliced into many pieces.

A circle is essentially a polygon with infinite sides of infinitessimal length.

We can therefore imagine each slice as a triangle.

Those triangles can be rearranged into a parallelogram.

Use the dimensions of the parallelogram to ‘discover’ the formula for circle area.

**Solve**

Find the area of each circle in terms of pi:

1. \( \text{Area} = \pi \times 5^2 \)  
2. \( \text{Area} = \pi \times 5^2 \times \sin 50^\circ \)  
3. \( \text{Area} = \pi \times (1/2)^2 \)

**Solve**

Find the circumference of each circle:

1. \( \text{Circumference} = 4\pi \)  
2. \( \text{Circumference} = 8\pi \)  
3. \( \text{Circumference} = 24\pi \)
Determine the area of each labeled region below:
Express in terms of pi and/or rounded to the tenth.

1. 

- half-circle a = ______m$^2$
- sector b = ______m$^2$
- triangle c = ______m$^2$
- circle segment d = ______m$^2$

Vocabulary:
A segment is the region between a chord and the included arc.
A sector is a ‘slice’ between two radii.
An annulus is the region between concentric circles (see below).

Determine the area of each labeled region below:
Express in terms of pi and/or rounded to the tenth.
The diameter of the small inner circle is 4cm.

- piece a = ______cm$^2$
- piece b = ______cm$^2$
- piece c = ______cm$^2$
- piece d = ______cm$^2$
- challenge piece e = ______cm$^2$
Other Uses for Area:

Finding an altitude:
If you know the area of a figure, you can often find the length of an altitude.

Example: Find the length of $x$ in the right triangle below. Express your answer as a common fraction.

![Right triangle diagram]

Hard Practice:
Find each missing length.

1. Find Altitude CD.  
2. Rhombus  
3. Parallelogram

![Triangle and parallelogram diagrams]

Word Problems:
1. The area of a parallelogram is 400cm$^2$. If the sides of the parallelogram measure 16cm and 40cm, what is the length of the long altitude?

2. What is the altitude to the hypotenuse of a right triangle whose legs measure 20 and 21cm?

3. The short diagonal of a rhombus of side length 25cm is 14cm. Find the diameter of the inscribed circle in the rhombus. Express your answer as a mixed number in simplest form.
Using Pi and Radical Notation

Practice: Find the area of the following triangles. Leave answer in simplest radical form:

1. (equilateral)

![Equilateral Triangle](image1)

2. (isosceles right)

![Right Triangle](image2)

Practice: Find each shaded area. Leave answers in terms of Pi or in radical form.

1. (large radius = 7cm)

![Octagon](image3)

2. (hexagon is regular)

![Hexagon](image4)

Find each shaded area. Leave answers in simplest radical form and/or in terms of Pi.

1. (octagon is regular)

![Octagon](image5)

2. (congruent circles have 6in radii)

![Circles](image6)
Using Pi and Radical Notation

Practice: Find each shaded area. Answers should be in simplest radical form and/or in terms of Pi where applicable.

1. _______
2. _______
3. _______
4. _______
5. _______
6. _______
7. _______
8. _______

note:
8.5 is a close approximation.
Can you find the real side length of the square in this diagram?
Geometry

Challenge set:
Find the shaded areas in terms of Pi, using radical notation.
Determine the area of each shaded region below:
Express in terms of pi and/or rounded to the tenth.

1. (parallelogram)

2. (pentagon is regular)

3. (octagon is regular)

4. (pentagon is regular)

5. (circles and squares)

6. (hexagon has side length 4)
Practice

Find the area of each numbered figure below.

Find the area of each. You may need to use the Pythagorean Theorem.
Answers should be rounded to the hundredth or in terms of pi.

1. (circle) _________
2. (????) __________
3. (segment) ________
4. (sector) __________
5. (????) __________
6. (partial annulus) __________

Find the area of each numbered figure below.
The figure below is a square. You may need to use the Pythagorean Theorem.
Round to hundredth (or leave answers in radical form).

7. (triangle) __________
8. (triangle) _____________
9. (triangle) _______________
10. (trapezoid) ___________
11. (triangle) ___________
12. (trapezoid) ___________
**Geometric Probability**

**Review:**
Find the labeled area in each diagram. Round decimal answers to the hundredth.

1. (figure is a square)  
   ![Diagram of a square with labeled area A]

2. (radius=2cm)  
   ![Diagram of a circle with labeled area B and C, C is a sector of 135°]

3. (radius=4cm)  
   ![Diagram of a circle with labeled area B and C, C is a sector of 135°]

**Geometric Probability:**
Use your answers from above to determine WHAT PERCENT of the figure is shaded. Round percents to the tenth.

1. (figure is a square)  
2. (radius=2cm)  
3. (radius=4cm)  

**Geometric Probability:**
Involves finding area ratios, usually represented as percents. For example:
Concentric circles have radii of 3 and 4 inches. If a random point is selected within the larger circle, what is the probability that it will be within the smaller circle as well? Express your answer as a fraction and as a percent.
**Geometry 8.6**

**Practice:** Find the percent of each shaded area below (to the tenth):

- **First Diagram:**
  - A circle with a radius of 6 cm.

- **Second Diagram:**
  - Four circles with a radius of 3 cm each.

- **Third Diagram:**
  - Fourteen circles with a radius of 4 cm each.

**Practice:**

What percent is shaded?

(The circle is inscribed in the large square and circumscribed about the large square).

**Practice:**

In the diagram, AB = 40 cm, BC = 20 cm.

What is the area of the shaded region? 
(in terms of \(\pi\))

What percent of the circle is shaded?

**Practice:**

Congruent semicircles AEB and BDC overlap semicircle ABC.

The radius of the large semicircle is 1.

What is the area of the shaded region?

What percent is shaded?
Geometric Probability

A common problem on EOC testing involves throwing darts at an imaginary dartboard, covered with a variety of geometric shapes.

1:
A carnival game involves throwing darts at the board below. You are just good enough to hit the board every time, but your aim is not good enough to control where you hit (you hit a random point on the board every time). What is the probability that you will hit each of the areas below?

Small circle radius: 4cm  Large radius: 10cm  Triangle sides: $4\sqrt{3}$ cm

Give answers as percents rounded to the tenth.

P(1 point) _________ or _____%  
P(2 points) ________ or _____%  
P(3 points) _________ or _____%  
P(9 points on three darts) __________ or _____%

Practice:
Determine the probability of hitting each region with one random dart:

Circle Radii = 2in  
You must use the Pythagorean Theorem to find the sides of the big square.

Give answers as fractions and percents rounded to the tenth.

P(A) _________ or _______%  
P(B) _________ or _______%  
P(C) _________ or _______%  
P(D) _________ or _______%  
P(B or D) __________ or _______%
Geometric Probability

Practice:
Determine the probability of hitting each region with one random dart:

Circle Radii = 4in
Give answers as fractions and percents rounded to the tenth.

P(1 point) \(\frac{\text{?}}{4}\) or \(\text{?}\)%
P(2 points) \(\frac{\text{?}}{4}\) or \(\text{?}\)%
P(5 points) \(\frac{\text{?}}{4}\) or \(\text{?}\)%
P(0 points) \(\frac{\text{?}}{4}\) or \(\text{?}\)%
Add all four answers = \(\text{?}\)%

Practice:
Determine the probability of hitting each region with one random dart:
The octagon is regular.

Octagon Perimeter: 16m
Give answers as fractions and percents rounded to the tenth.

P(A) \(\frac{\text{?}}{16}\) or \(\text{?}\)%
P(B) \(\frac{\text{?}}{16}\) or \(\text{?}\)%
P(C) \(\frac{\text{?}}{16}\) or \(\text{?}\)%
P(D) \(\frac{\text{?}}{16}\) or \(\text{?}\)%
**Surface Area**

**Surface Area** is the sum of the areas of all faces which enclose a solid. It is useful to be able to recognize some of the nets which represent solid figures.

What solid does each figure above represent? Draw a sketch of each below.

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**Formulas:**

Sketch each figure below and write a formula for its surface area:

- **Rectangular Prism**
  (box)

- **Square Pyramid**
  (like in Egypt)

- **Cylinder**
  (soda can)

- **Cone**
  (sugar cone not a waffle cone)
Surface Area

Determine the surface area of each solid below:
Round all answers to the hundredth.

1. A = ______
2. A = ______
3. A = ______
4. A = ______
5. A = ______

6. A = ______

7. A = ______

Circle radius: 12ft
Pentagon sides: 3ft
Pentagon apothem: 2ft

(Don’t forget the surfaces ‘inside’ the hole.)
Surface Area

Surface areas will be generally combinations of the following types.
Find the area of each. Leave answers in radical form and/or in terms of Pi.

1. **Prism**
   - Height = 11 cm
   - Base = 7 cm
   - Side = 10 cm

2. **Cylinder**
   - Radius = 4 in
   - Height = 3 in

3. **Pyramid**
   - Base = 8 cm
   - Height = 3 in

4. **Cone**
   - Slant Height = 10 ft
   - Radius = 6 ft
   - Height = 10 ft

**Combinations:**
Find the surface area of each combination below.
Leave answers in radical form and/or in terms of Pi.

1. **Prism with a Cylindrical Hole (diameter = 2 ft)**
   - Height = 3 ft
   - Base = 3 ft

2. **Truncated Cone (Frustum)**
   - Height = 10 ft
   - Radii = 6 ft and 12 ft
Determine the geometric probability of striking each labeled region in the diagrams below:
Answers should be rounded to the tenth of a percent.

Determine the surface area or each figure below:

- cone:
  - slant height: 12 in
  - radius: 5 in

- circle:
  - diameter: 3 ft

- regular octahedron:
  - 8 sides, all equilateral triangles
  - edge: 10 cm
Determine the shaded area of each figure below. Leave answers in terms of pi and in simplest radical form where applicable.

1. __________
2. __________
3. __________
4. __________
5. __________
6. __________
7. __________
Practice Test

What is the geometric probability of striking each shaded region below? Round to the tenth where necessary.

8. __________

9. __________

10. __________

Find the surface area of each.

11. Base = _______
12. Total = _______

13. Top + Bottom = _______
14. Total = _______