Trigonometry

The three primary Trigonometric Ratios are **Sine, Cosine, and Tangent.**
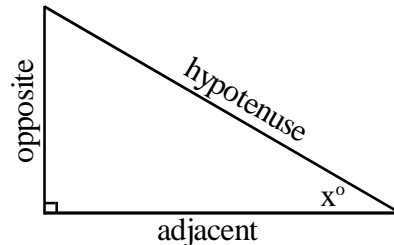
As we learned previously, triangles with the same angle measures have proportional sides. If you know one angle in a right triangle, you can use sin, cos, and tan to find the ratio of the lengths of its sides.

Sine, Cosine, and Tangent
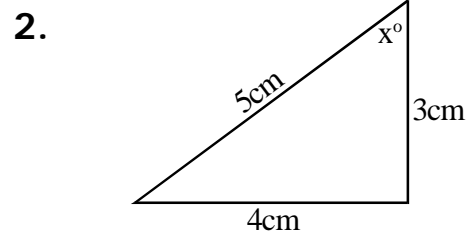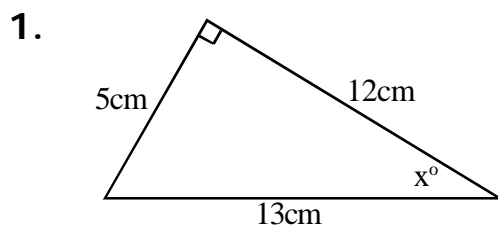
Sin x° = $\dfrac{\text{opposite}}{\text{hypotenuse}}$

Cos x° = $\dfrac{\text{adjacent}}{\text{hypotenuse}}$

Tan x° = $\dfrac{\text{opposite}}{\text{adjacent}}$

Find sin x, cos x, and tan x in the right triangles below:

1.

5cm   12cm   x°   13cm

Sin x° =   Cos x° =   Tan x° =

2.

x°   5cm   3cm   4cm

Sin x° =   Cos x° =   Tan x° =

If you know an angle, you can use a calculator to find the ratios:

1. sin 34° =   **2.** cos 55° =   **3.** tan 18° =

Use a calculator to find the missing lengths below:

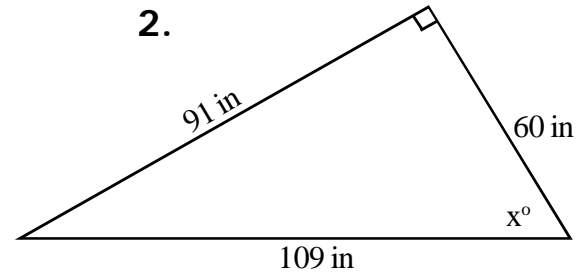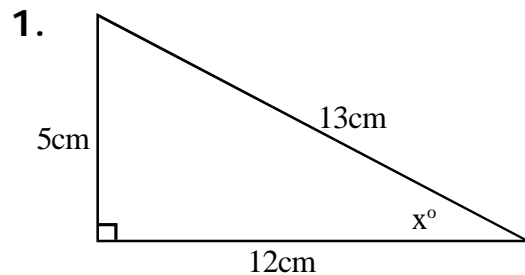**1.**

5cm   y   27°   x

2.

a   b   58°   22cm
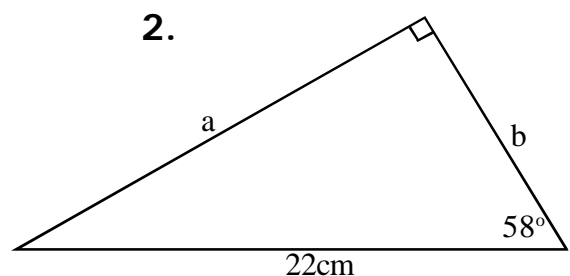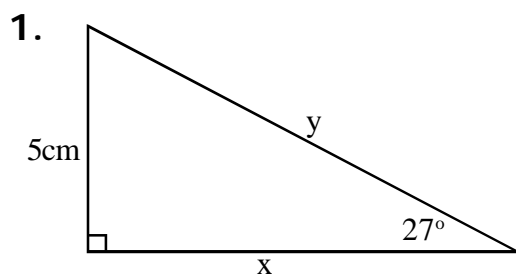
Trigonometry

You can use trigonometric inverses to find an angle based on a ratio:
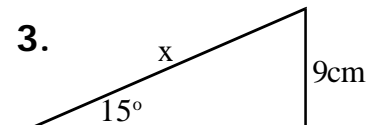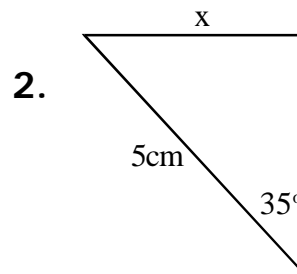
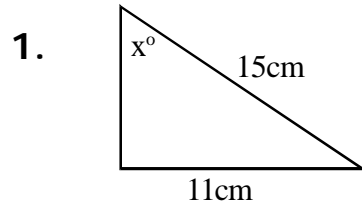**Sin, Cos, and Tan** turn an angle into a ratio.
Sin^{-1}, Cos^{-1}, and Tan^{-1} turn a ratio into an angle measure.

Example: Find the measure of angle x in each triangle below:
(make sure your calculator is set to degrees not radians)

1.

13cm
5cm
x°
12cm

2.

91 in
60 in
x°
109 in

Practice: Find the missing information in each right triangle below:
(round to the hundredth)

1.

x°
15cm
11cm

2.

x
5cm
35°

3.

x
15°
9cm

Practice: Find the apothem in each regular polygon below:

1.

8cm

2.

10cm

3. Find the sides.

6cm
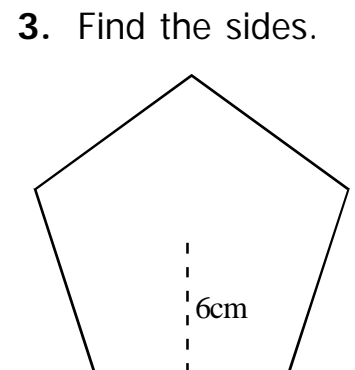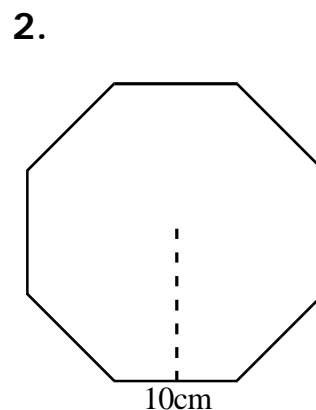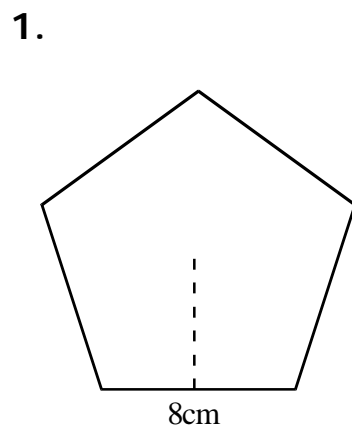
Trigonometry Practice: Basics

Geometry

Practice: Solve for x in each of the diagrams below.

1.

x

85cm

77cm

2.

4cm

5cm

x

3.

4cm

34°

x

1. _____

2. _____
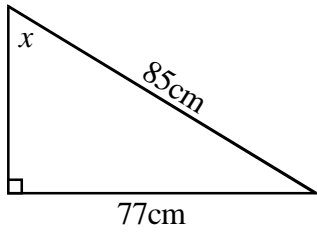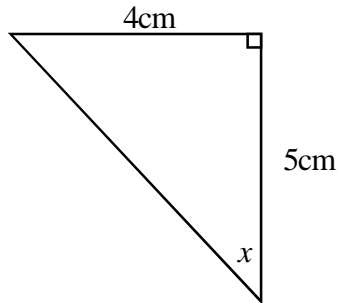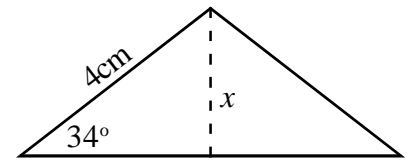
3. _____

Practice: Solve each. Round decimal answers to the tenth unless noted otherwise.

4. What is the measure of the smallest angle in a 20-21-29 right triangle?
Round your answer to the tenth of a degree.

4. _____

5. A right triangle has a short leg measuring 10cm and one acute angle measure
of 55°. What is the length of the hypotenuse?

5. _____

6. What is the length of the apothem of a nonagon whose sides measure 4cm?

6. _____

7. Pentagon ABCDE has an edge length of 6cm. What is the length of segment AC?

7. _____

Trigonometry Practice: Basics Geometry

Practice: Solve each. Round decimal answers to the tenth unless noted otherwise.

8. What is the side length of a 12-gon whose apothem is 20cm long?

8. _____

9. Find the area of a 15-gon whose sides measure 2cm.

9. _____

10. A decagon of side length of 10cm contains an inscribed circle. What is the area of the inscribed circle in terms of *tan* and pi? ex. $\pi(3\tan 75)^2$

10. _____

11. What is the altitude h of the triangle below?

29°

h 3cm

33°
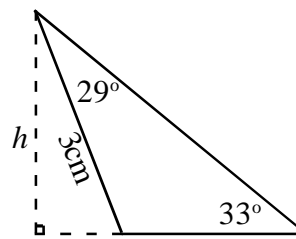
11. _____

12. A regular polygon has an apothem of about 101cm and a side length of 32 cm. How many sides does it have?

12. _____

Trigonometry

The three trigonometric ratios have a variety of uses.

If you can measure the angle of elevation between yourself and an object, you can find its height given the distance you are from it horizontally. If you know its height, you can determine the distance you are from it.

Example:
You are driving up long mountain road at a 7 degree angle of incline. After driving 16 miles you reach the peak. How high is the mountain? (5,280 ft/mile)

Practice: Round to the tenth.
1. You can view the top of the Empire State Building at a 12 degree angle from where you are in Central Park. You know that the top of the building is 1,472 feet. How many feet must you walk to reach the base of the building?

2. The wire that supports a very tall television antenna is attached 45 feet from the base of the antenna, and makes a 81.5 degree angle with the level concrete pad which anchors it. How tall is the antenna?

Practice: Round to the tenth.
1. The minimum slope for a gutter is 1/8″ per foot. What is the angle of the slope of the gutter?

2. A wheelchair ramp can have a maximum slope of 4°. If a ramp must climb to the top of a six-foot platform, what is the minimum amount of space left in front of the platform for the ramp?

Practice: Solve for x.

1.


13m

2.

36°

x

5cm

3.

5cm

x

40°
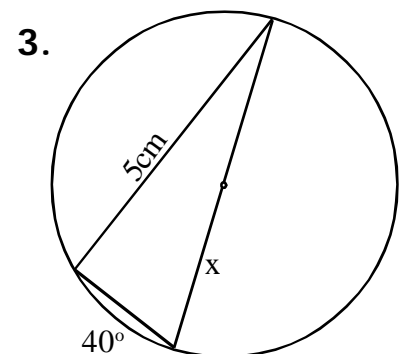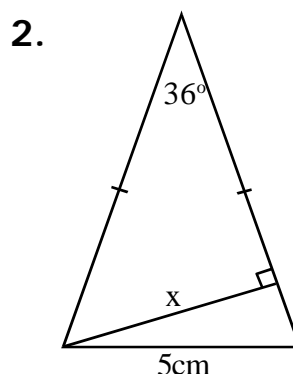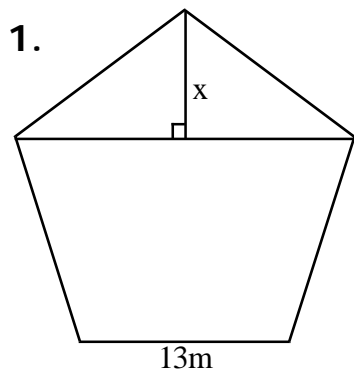
Trigonometry

Practice: Solve each.

1. The legs of a right triangle are 3.9 and 8 inches long.
    What are the measures of the angles?

2. An isosceles triangle has 20° base angles and a 4-inch base.
    Find its area.

3. You can view the top of a distant mountain at an angle of 3 degrees
    above horizontal. If the mountain peak is 8,014 feet above your
    current elevation, how many miles are you from the peak of the
    mountain?

Practice: Solve each.

1. In right triangle ABC, Sin A = 28/53. What is Cos A?

2. What are the coordinates of point B in terms
   of the angle below?
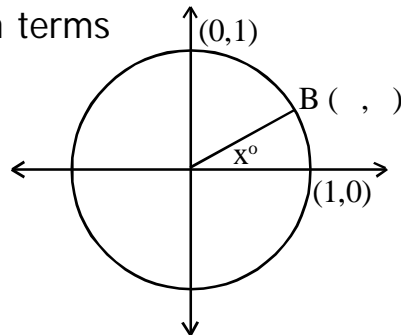
3. To fly to Brighton Airport, you can fly 215 miles at a heading that is
    exactly 7 degrees east of due north. Instead, you decide to fly
    due north until you are at the same latitude as Brighton, then
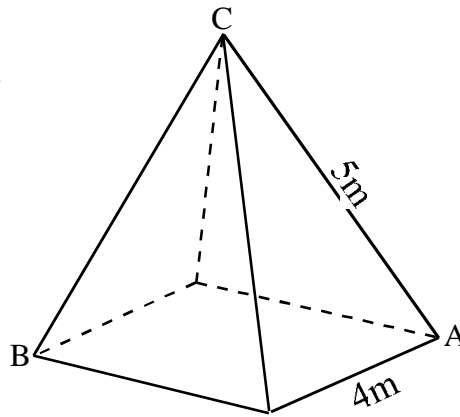   due east to land at the airport. How many miles will you fly?

Trigonometry Practice

Practice: Solve each. Round decimal answers to the hundredth.

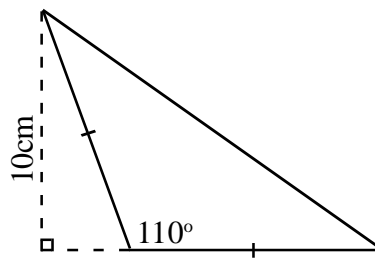**1.** A right cone has a 4-inch radius and a slant
angle of 55°. Find its volume.
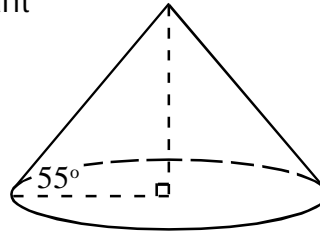
1. _____

2. A square based pyramid has
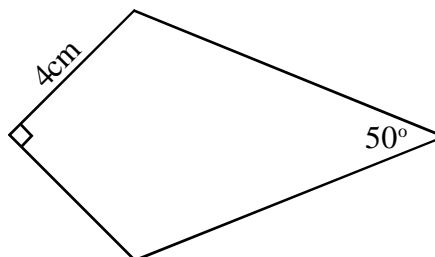edges as shown. Find the
measure of angle ABC.

2. _____

3. Find the area of the following
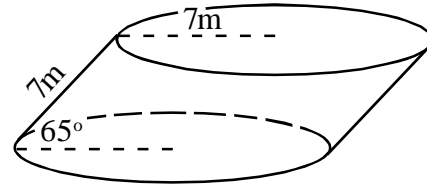isosceles triangle:

3. _____

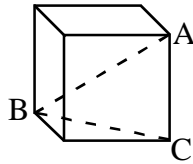**4.** Find the area of Kite ABCD:

4. _____

Trigonometry Practice

Practice: Solve each. Round decimal answers to the hundredth.

5. An oblique cylinder is slanted at an angle of 65°.
Find its volume if its radius and slant height
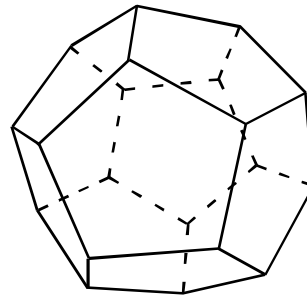are each 7m.

7m

7m

65°

5. _____

6. Find the measure of angle ABC in the cube below.

A

B

C

6. _____

7. What is the surface area of a dodecahedron of edge length 2?
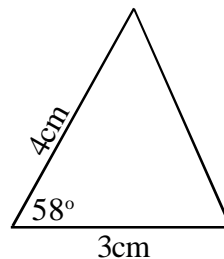(12 pentagonal faces)

This answer may be left
in trig. notation.
ex. 8 sin(35°)

7. _____

8. Find the area:
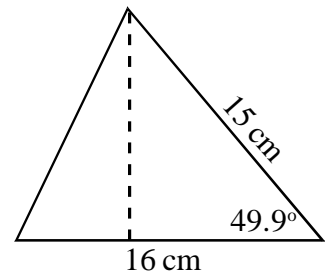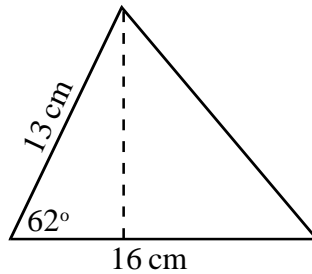
This answer may be left
in trig. notation.

4cm

58°

3cm

8. _____

Trigonometry

The trigonometric ratios can be used to solve problems involving all types of triangles.
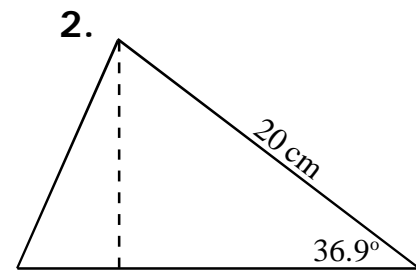
Find the area of each triangle below:

1. Find the altitude using sine.
2. Multiply by the 1/2 the base.

13 cm
62°
16 cm

15 cm
49.9°
16 cm

You can always find the area of a triangle using two sides and the included angle by finding the altitude.

Determine the altitude of the triangles below:

1.

13 cm
67.4°

2.

20 cm
36.9°
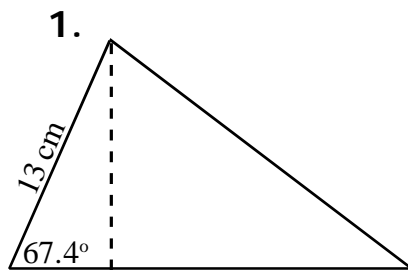
Use the same technique to find the altitude in each triangle below in terms of the sides and angles labeled:

1.

b
h
A

2.

a
h
B
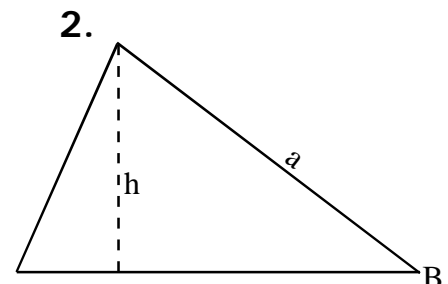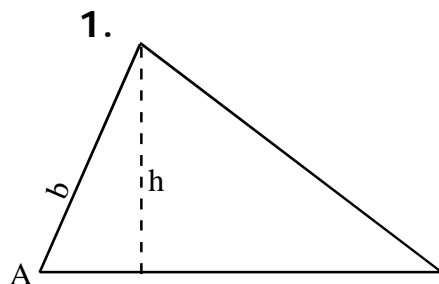
$b \sin A = h$ $a \sin B = h$ therefore $a \sin B = b \sin A$

Some simple Algebra gives us the **Law of Sines:** $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$

b   a
A   B
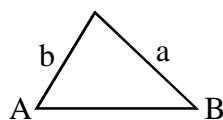
Trigonometry

Use the Law of Sines to find each missing length x:
Round to the tenth.

1.

63°

27

81°

x

2.

39°

x

102°

17
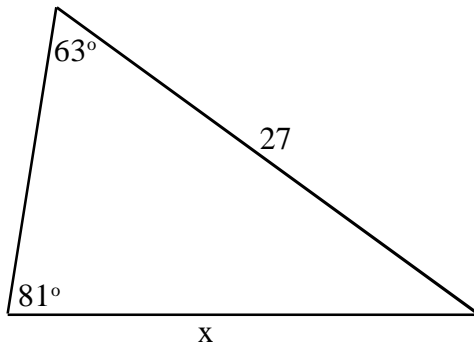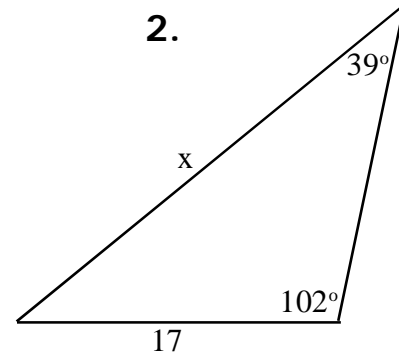
You can use the law of sines to find missing angles as well.
Solve for sin x and then use sin⁻¹ to determine the missing angle.
This ONLY WORKS IF YOU KNOW THE ANGLE IS ACUTE.
Round to the tenth.

1.

65°

16

$x°$

25

2.

$x°$

26

102°

17

Find the area of each figure below:

1.

20m

112°

9m

2.

12m

102°

13m

18m

59.6°

21m
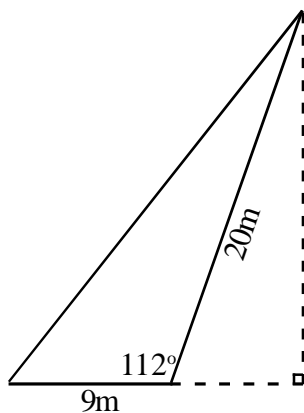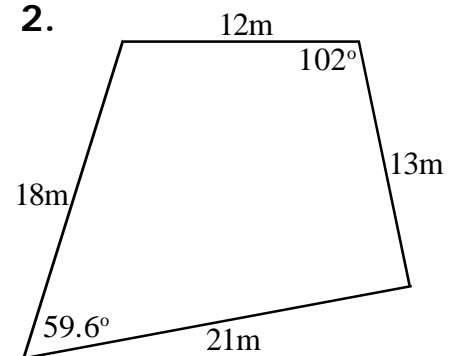
Extended Law of Sines

We can use the **Law of Sines** to determine the diameter of the circumscribed circle for any triangle... and the proof is easy!

In the diagram below, which two angles are equal?

What is $sin\ X$?

What is $sin\ A$?

Can you write an expression for d ?
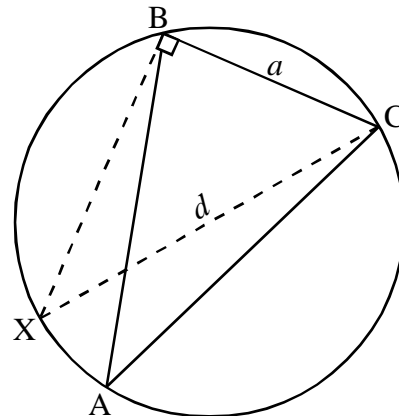
The Extended Law of Sines: $2R = \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Another area formula for triangles:
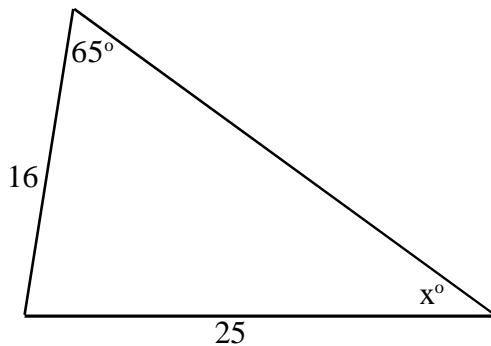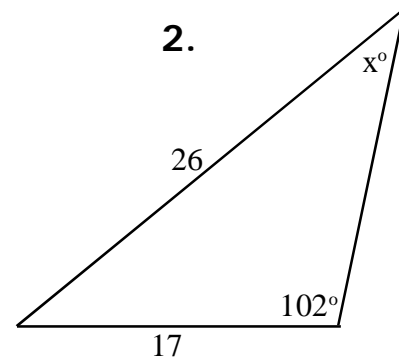
We know that $2R = \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ and $A = \dfrac{1}{2}ab\sin C$.

Solve for $\sin C$ (in terms of R) in the first equation, substitute it into the second, and find a formula for the area of a triangle based on its sides and its circumradius R.

Given the sides and the circumradius of a triangle, its area $A = \dfrac{abc}{4R}$.

Extended Law of Sines, Area.

Practice:

Find the area of each circle. Round to the tenth.

1.

B

83° 6cm

C

37°

A

2.

B

123°

A

C

2cm
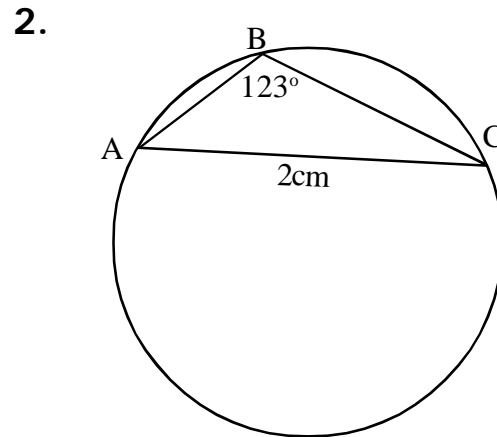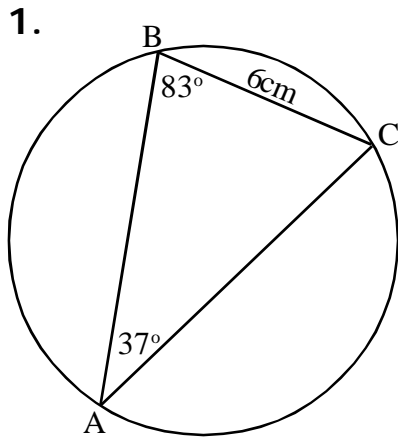
Practice:

Find the area of each triangle. Round to the hundredth. No cheating with Heron's Formula.

1.

B

6cm

A

C

5.1cm

10cm

9cm

2.

B

23cm

27cm

45cm

C

A

28.6cm
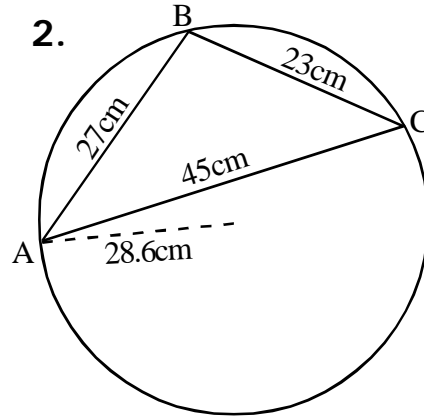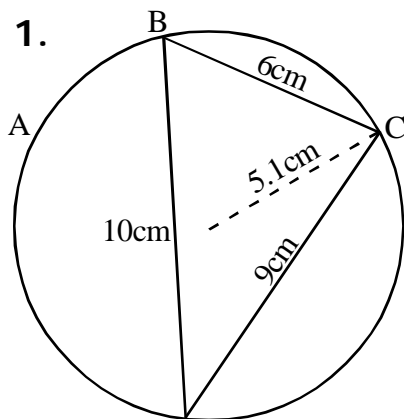
Practice:

Find the missing length:

B

3cm 120° 5cm

A

C

x cm

8cm
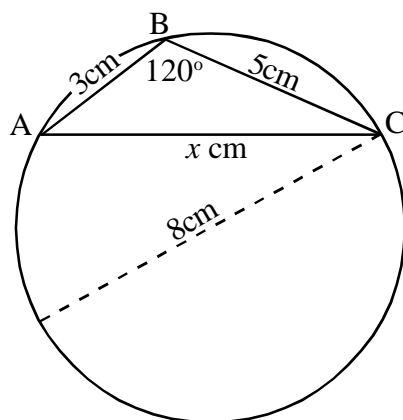
Law of Cosines

If two sides of any triangle are known, along with the angle opposite the missing side, the **Law of Cosines** can be used to find the missing length.
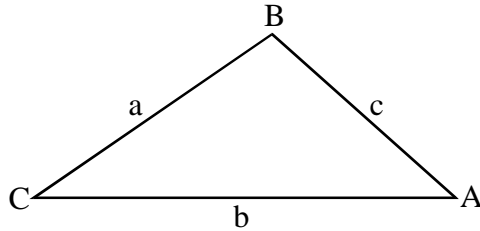
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Deriving the **Law of Cosines**:
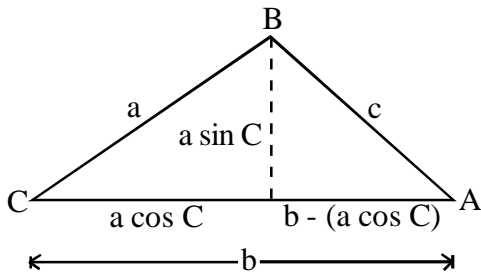
Pythagorean Theorem:

$c^2 = (a \sin C)^2 + [b-(a \cos C)]^2$

$c^2 = a^2(\sin C)^2 + b^2 - 2ab(\cos C) + a^2(\cos C)^2$

$c^2 = a^2[(\sin C)^2 + (\cos C)^2] + b^2 - 2ab(\cos C)$

$c^2 = a^2 + b^2 - 2ab \cos C$

Using the **Law of Cosines**:
Works with two sides and the included angle to find the missing side, or three sides if you are looking for an angle measure.

Practice:
Find the length of x and the missing angle y below:

6m   x   68°   7m

13in   27in   y°   26in
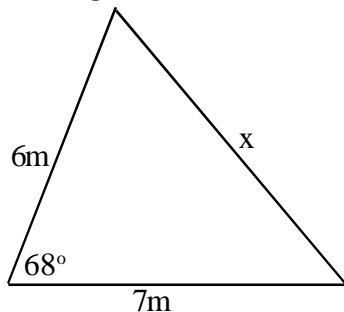
7.32m
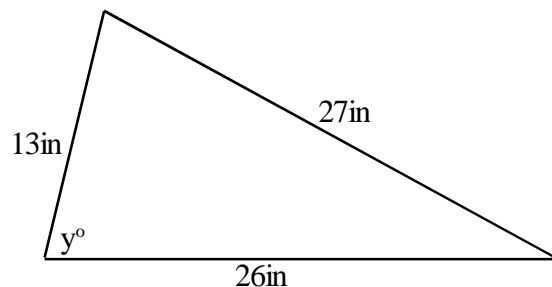
80.12°

Law of Cosines

Practice:
Find the length of each missing side.

1.

16m   x

59°

23m

2.

7m

a

29°

5m
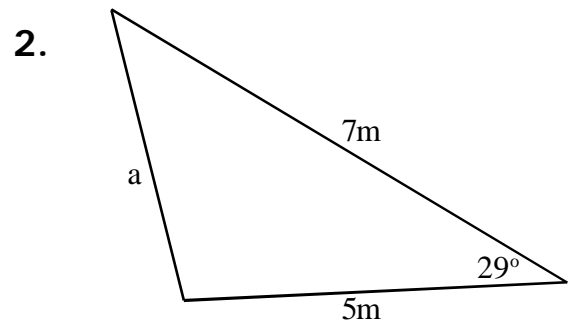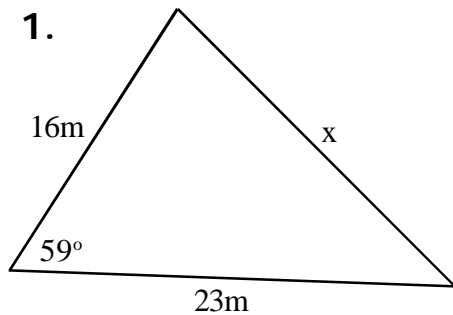
Practice: Combine the Law of Sines with the Law of Cosines to find all missing sides and angles:

107°

19m   23m

$y°$

$z°$

x
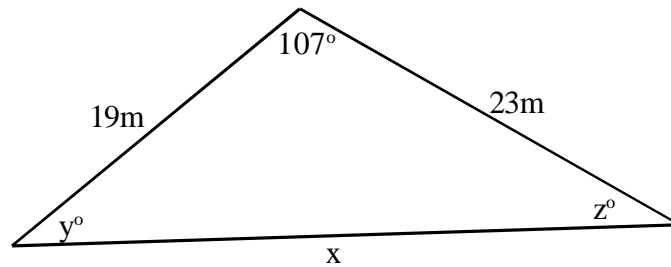
Practice: Solve.

1. What are the angle measures in a 3-7-9 triangle?

2. In regular pentagon ABCDE, AB = 8. Find the length of diagonal AC.

3. What is the length of the altitude perpendicular to the longest side
of a 6-7-8 triangle?

4. Find the area of a 2-3-4 triangle using the trigonometry we
have learned in class.
(no cheating with Heron's Formula if you know it).

Applying the Law of Cosines

Forces are often represented by **vector diagrams**. The resultant vectors can be calculated using **Trigonometry.**

First, apply the Law of Cosines to determine the diagonals of each parallelogram below:

1.

2.
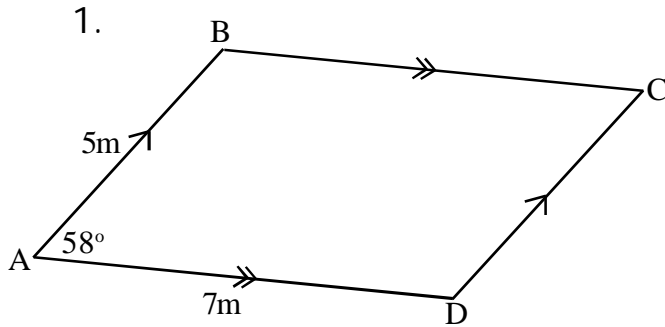
AC = _____

BD = _____
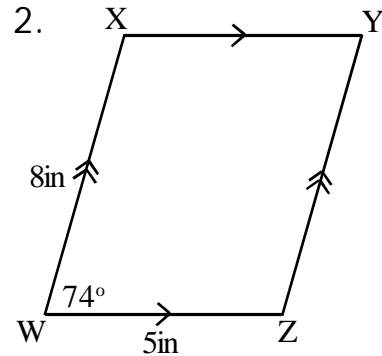
WY = _____

XZ = _____

A brief note about heading/bearing:
In navigation, $0°$ is considered due north and bearing angles are measure clockwise ($90°$ is due east, $180°$ is south, $225°$ is SW, etc.)

Example:
A plane is headed at a bearing of $55°$ traveling at an airspeed of 140 mph into a 60mph wind coming from $0°$ (due north). What is the resultant ground speed and heading of the plane?

Practice:
1. A plane is heading southeast ($135°$) with an airspeed of 180 mph with a 35mph tailwind coming from the west ($270°$). What is the ground speed and heading of the plane?

2. To cross a river, a canoe is aimed $20°$ left of directly upstream. Matt and Maria can paddle the canoe 5mph on flat water, but are paddling against a 3mph current. What direction and speed will the canoe cross the river?

3. Greg maintains a constant speed of 10mph on his bicycle at a $320°$ bearing for 50 minutes. He changes direction and travels with a $15°$ bearing for 22 minutes. Finally, he rides directly back to his starting point. How long does it take him to get back, and what should his bearing be?

Practice:

3. To find a lost pirate treasure, a map directs you to sail 6km at a 40 degree heading, then 4km at a 160 degree heading. You would prefer to sail directly to the treasure. What distance and heading should you sail?

4. To get from Pueblo to San Rafael, you fly 40 miles at a 44 degree heading. To get from Pueblu to San Marco, you fly 12 miles at a 305 degree heading. What distance and heading would you use to fly from San Marco to San Rafael?

Applying the Law of Cosines

Practice:

1. Karen is flies her small plane 70 miles per hour at a 200 degree heading, but the wind is blowing from the southeast at 20 miles per hour. If she flies for one hour at this speed and heading, and the wind does not change, how far will she be from where she began?

2. Two beams are supported by a column as shown, with the force applied by each column given. What load must the column be able to support in order to hold the two beams?
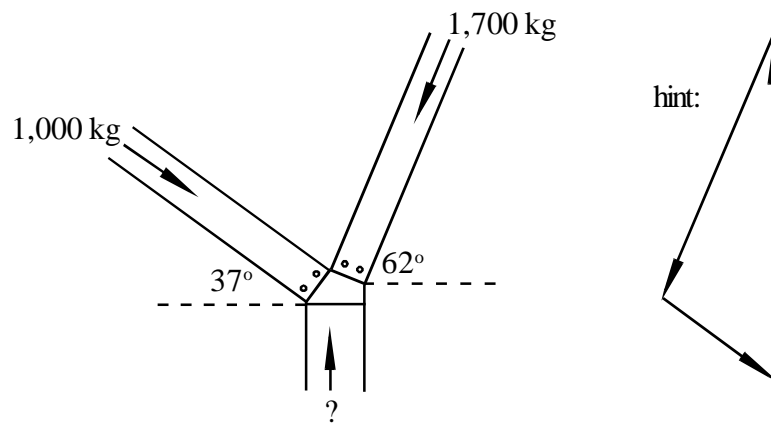
Trigonometry Practice Quiz Geometry 12.3

Solve: Find each missing length. Round to the hundredth.

1.
90cm
x
27°

2.
18 m
x
70°

3.
5 m
x
58°
3m
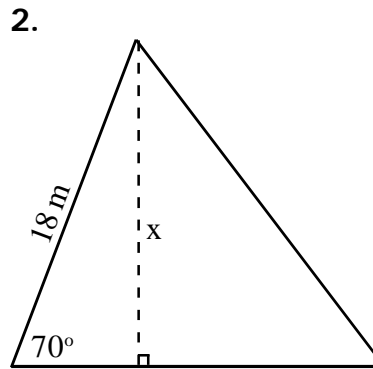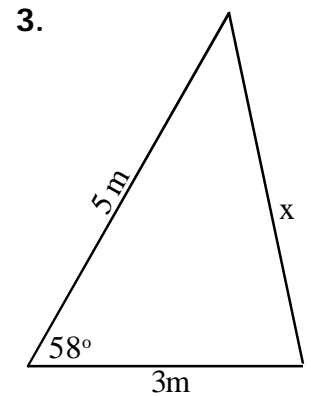
1. x = _____   **2. x = _____**   **3. x = _____**

Solve: Find each missing angle. Round to the hundredth.

4.
x°
23cm
16cm

5.
x°
25cm
18cm
23cm

6.
10cm
67°
24cm
x°

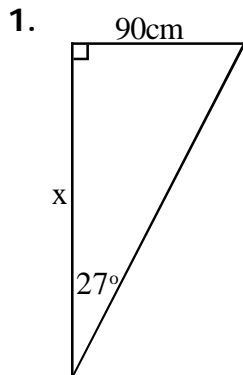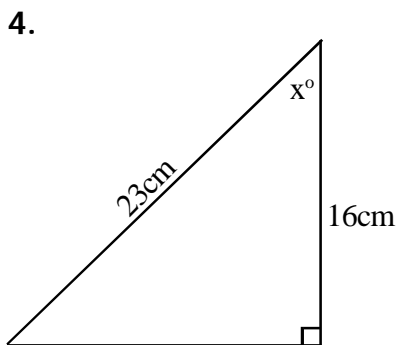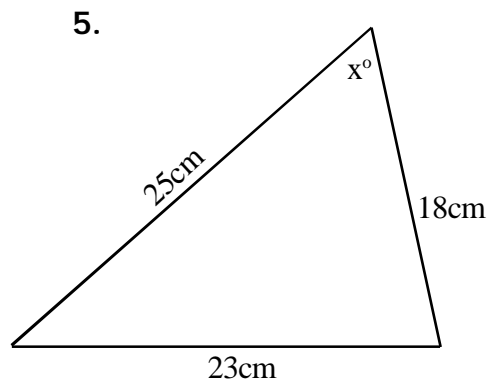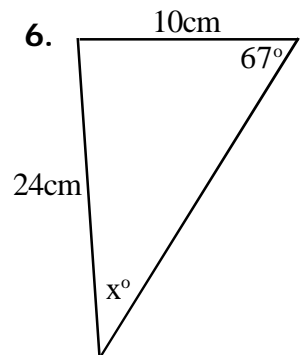**4. x = _____**   **5. x = _____**   **6. x = _____**

Solve:

7. A triangle of side lengths 6, 7, and 11 cm is inscribed within a circle.
What is the area of the circle?

7. _____

8. The slope of a ramp is 5°. If the ramp is 26 feet long, how much
higher is the top of the ramp than the bottom of the ramp?

8. _____

Trigonometry Practice Quiz

Solve: Find the perimeter. Round decimal answers to the hundredth.

10.

18m

10. P = _____

Solve: Find the area of each:

11.   **12.**

10cm   40°

12cm   15cm

80°
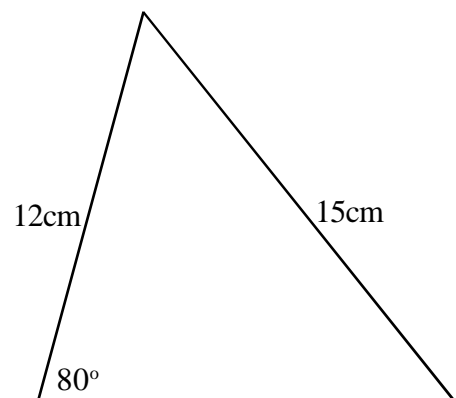
11. A = _____   **12. A = _____**

Additional Trig. Applications   Geometry 12.5

Coordinate Applications:

1. A line connects points (-1, -5) and (5, 8), forming an acute angle when it crosses the x-axis. What is the measure of the angle?

2. What is the measure of the acute angle formed when the line formed by the equation $y = -\dfrac{2}{7}x + 3$ crosses the y-axis?

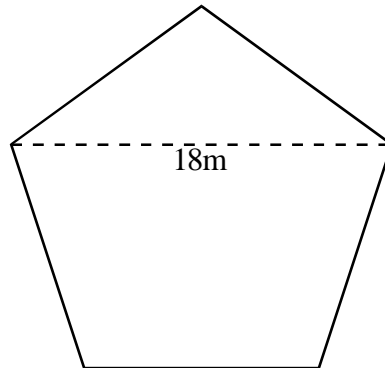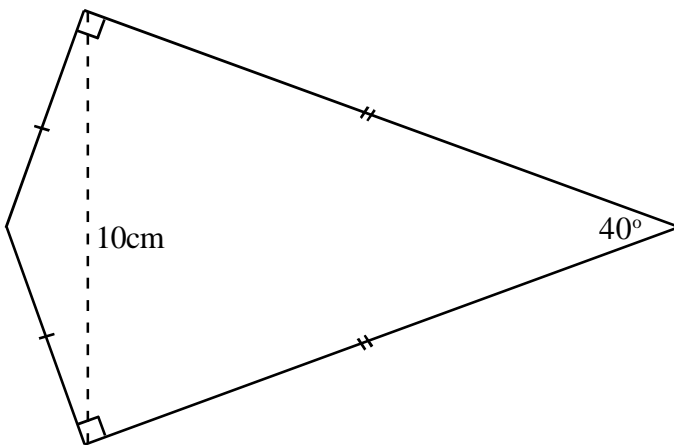**3.** A line passes through the origin forming a 26.566° angle in the first quadrant with the x-axis. What is the slope of the line?

Area Applications:

4. The sides of a triangle measure 5, 6, and 9 centimeters. What is its area?

5. A regular heptagon has 9-inch sides. What is its area?

6. Martina is renting an irregular office space by the square foot and feels she is being overcharged. She measures the walls of the office, which are 15, 28, 45, and 50 feet long. The only right angle is between the 28 foot wall and the 45 foot wall. She is being charged for 1,000 square feet. Is she being overcharged? If so, by how much?

Additional Trig. Applications   Geometry 12.5

Similarity Applications:

7. The triangles below are similar. What is the perimeter of the entire figure? (round the tenth)

6 ft

122°

3 ft
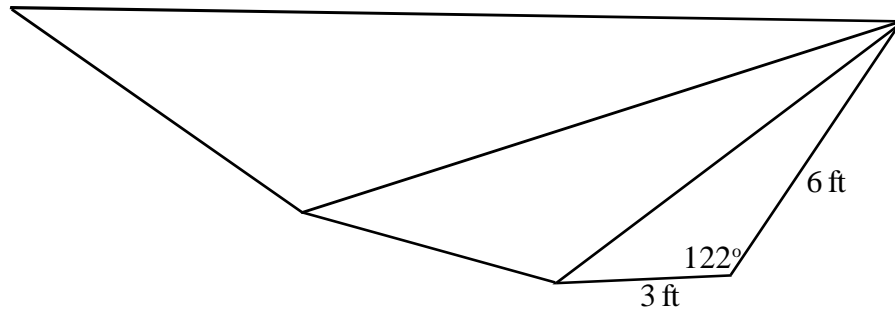
8 (challenge). Inscribed trapezoid DEFG is similar to trapezoid ABCD. What is the measure of angle FCG?

D   G   C

2cm

E   5cm

F

A   4cm   B

Navigation Applications:

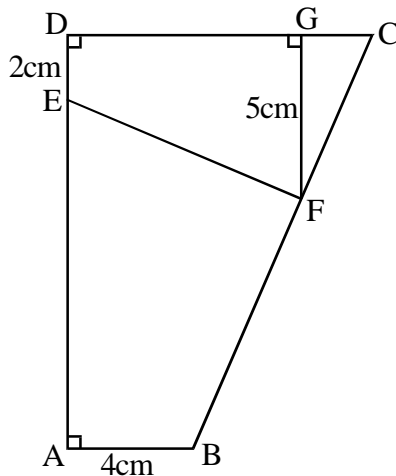**9.** On a map, you draw lines connecting JFK airport and Chicago O'Hare Airport to RDU airport. The angle between the two lines is 63°. It is 500 miles from RDU to JFK, and 850 miles from RDU to O'Hare. How many miles is it from JFK to O'Hare? (round to the mile)

10. Two boats leave port at 9am. The first boat travels northeast at 22 mph, while the second boat travels west at 15 mph. After two hours, they decide to meet by turning directly towards each other (each boat continues to travel at its original speed). What time do the boats meet? (round to the minute)

Trigonometry Practice

Solve: Round decimal answers to the hundredth.

1. What acute angle is formed where the equation $y=3x+7$ crosses the x-axis?

1. _____

2. Find the acute angle formed by the intersection of the two lines below:

$$y=2x+5 \qquad y=-\frac{4}{5}x+1$$

2. _____

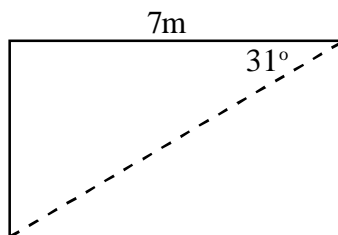**3.** Triangle ABC is formed by connecting the following coordinates on the plane:

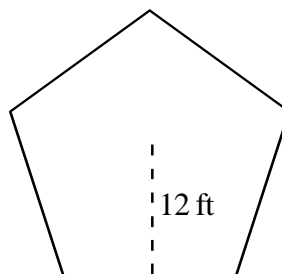A: (4, 7)   B: (-3, 1)   C: (3, -5)

Find m∠ABC.

3. _____

Practice: Find the area of each figure below.

4. (rectangle) **5.** (regular pentagon) **6.** (regular nonagon)

7m
31°

12 ft

8 m
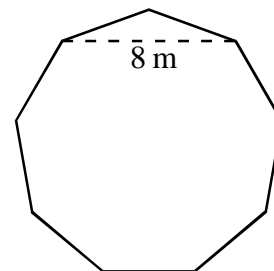
4. A = _____   **5. A =** _____   **6. A =** _____

Solve:

7. Kite ABCD has congruent angles B and D, with angle A = 50º and angle C = 110º. Diagonal AC is 14 meters long. What is the kite's perimeter (to the hundredth).

7. _____

8. Quadrilateral ABCD has right angle B, and sides AB = 3cm, BC = 4cm, CD = 5cm, and AD = 6cm. What is the measure of angle D?
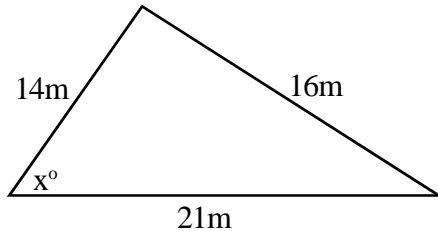
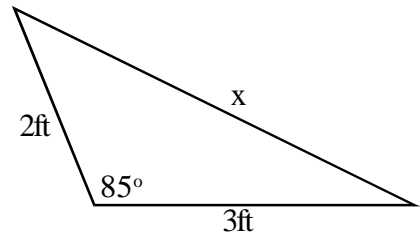8. _____

Trigonometry Review

You should recognize when to use the Law of Sines and when to use the Law of Cosines in each problem below.

Which would you use: The Law of Sines or the Law of Cosines? Do not solve.

1.

14m   16m

xº

21m

2.

x

2ft

85º

3ft

3.

27ft

xº

22ft

62º

4.

x

42º

35º

22ft
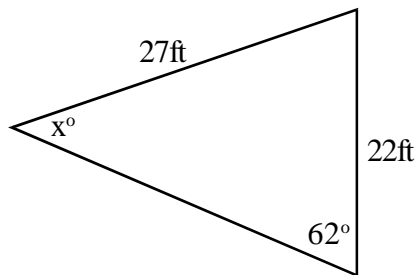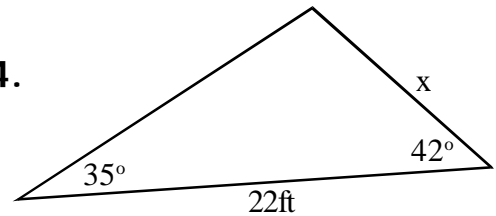
Practice:

1. What is the measure of the acute vertical angles formed when the equations y=2x and y=3x-1 cross?

2. What is the area of a 10-11-12 triangle?

3. Patrick and Ricky are at the Pentagon (In Washington D.C.). Standing at one corner with their backs on adjacent walls, each walks straight out for 90 feet. How far apart are they when they stop?

4. What is the short altitude of a 35-55-90 right triangle whose hypotenuse is 22 meters long?

answers: 8.13º,   51.52, 105.80ft, 10.34m

Practice Test: Trigonometry Geometry 12.5

Solve: Round to the hundredth unless noted otherwise.

1. What is the smallest angle measure in a 4-5-6 triangle?

1. _____

2. The hypotenuse in a right triangle measures 14cm. One of the two acute angles measures 34°. What is the perimeter of the triangle?

2. _____

3. Trains have a very difficult time climbing steep inclines. If the maximum angle that a train can climb is 3°, how many miles long must the **tracks** be to climb 7,000 feet up over a mountain pass? Round to the nearest mile.

3. _____

4. The wire supports for a local television station antenna attach to the top of the tower and to the ground, 45 feet from the base of the antenna. You measure an 82° angle between one of the wires and the ground. How tall is the antenna?

4. _____

5. What is the altitude of a 5-7-8 triangle perpendicular to its longest side?

5. _____

Practice Test: Trigonometry   Geometry 12.5

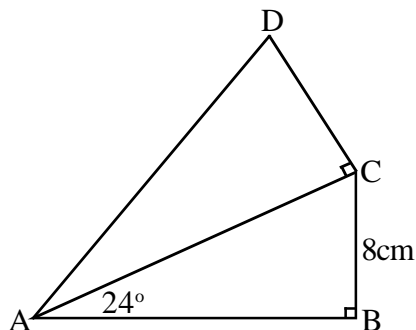## Solve: Round to the hundredth unless noted otherwise.

6. What is the area of an isosceles triangle whose base angles
measure 39° and whose base is 12.8 cm long?

6. _____

7. Point A: (7,3) Point B: (0,2) Point C: (9,-7)
Find m∠ABC.

7. _____

8. Triangles ABC and ACD below are similar. What is the
area of quadrilateral ABCD?

8. _____

9-10. Evan is searching the desert for buried treasure. The map he has says that
from where he is standing, he should walk 64 paces at a 25° bearing, then walk 25
paces at a 155° bearing. Instead, Evan is too lazy to walk that far (but not too lazy to
do the math).

9. How many paces will he have to walk to go directly to the treasure?
(round to the nearest pace)

9. _____

10. What bearing should he walk? (remember, 0° is due north, 90° is east, etc.)

10. _____