Polynomials, FOIL, and Factoring

You must be able to work with polynomials on the EOC:

Distribution and FOIL:
Rewrite each using distribution:

1. \(2x(x - 3)\)  
2. \(x - 5(x - 3)\)  
3. \((x - 5)(x - 3)\)

Practice:
Distribute. Simplify where possible.

1. \(x - 5x(7 - x)\)  
2. \((2x + 7)(x - 5)\)  
3. \((5x - 3)(5x + 3)\)

Practice:
Special Products.

1. \((7 + x)(7 - x)\)  
2. \((2x + 7)^2\)  
3. \((x - 3y)^2\)

Factoring: GCF.
Rewrite each by factoring the GCF:

1. \(2x^2 - 30x\)  
2. \(9x^2y^5 - 6x^5y^2\)  
3. \(4x^2 - 16\)

Factoring: Easy Ones.
Example: Factor.

1. \(x^2 - 3x - 35\)

Practice: Easy Ones.
Factor each.

1. \(x^2 + 4x - 5\)  
2. \(x^2 - 6x + 8\)  
3. \(x^2 - x + 7\)
Factoring: Special Products.
Example: Factor.

1. \(9x^2 - 42x + 49\)  
2. \(9x^2 - 64\)

Practice: Special Products.
Factor each.

1. \(4x^2 - 20x + 25\)  
2. \(121x^2 - 1\)

Factoring: Hard Ones (Magic Number).
Example: Factor.

1. \(2x^2 + x - 10\)  
2. \(15x^2 - 11x + 2\)

Practice: Magic Number.
Factor each.

1. \(7x^2 + 2x - 9\)  
2. \(6x^2 - 13x - 5\)

Factoring: Solving a quadratic by factoring.
Example: Solve for \(x\).

1. \((x - 3)(2x + 1) = 0\)  
2. \(x^2 - 11x + 30 = 0\)

Practice: Solve by factoring.
Solve for \(x\).

1. \(x^2 + 2x - 15\)  
2. \(9x^2 - 25\)
Direct Variation

Direct Variation is just Slope-Intercept Form (without the intercept).

If a problem states:  \( y \) varies directly as \( x \),
That means \( y = kx \) for some value \( k \).

\[ y = kx \]

\( k \) is called the constant of variation.
You can think of it as slope.

You can also say that \( x \) varies directly as \( y \): this means \( x = ky \)

Example 1:
When \( y = 6 \), \( x = 2 \), solve for \( x \) when \( y = -9 \) if \( y \) varies directly as \( x \).

Example 2:
The distance it takes to stop a moving train varies directly with the speed it is traveling. A train that is moving 50mph requires 10,000 feet to stop. How many feet will be required to stop a train moving 45mph?

Practice:
Solve each using direct variation. In each problem, \( y \) varies directly as \( x \).

1. When \( y = 8 \), \( x = 5 \). What is the constant of variation?
2. When \( x = 2 \), \( y = 7 \). Find \( y \) when \( x = 3 \).
3. The skid marks left by a vehicle can be used to determine the speed with which it was traveling. If an 18-wheeler leaves 200-foot skid marks, it was traveling approximately 60mph. How fast was an 18-wheeler traveling if it left 240-foot skid marks?

Proportional Reasoning can also be used for most of these problems:
Example 1:
The mass of an element varies directly as its volume. If 10cm\(^3\) of Carbon weighs 22.6 grams, how much will 12cm\(^3\) weigh?

a. Solve using a proportion.
b. Solve using direct variation.
c. What is the constant of variation?
d. What do we call the constant of variation in this problem?

Practice:
The amount of stretch of a rubber band varies directly as the force applied to it. If a 10-gram weight stretches a rubber band by 8cm, how much will a rubber band stretch when weighted with a 14-gram weight?
Distance/Midpoint Parallel/Perp.

The distance between coordinates on the plane can be found using the Pythagorean Theorem.

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

The midpoint between two points \((x_1, y_1)\) and \((x_2, y_2)\) is found by averaging the \(x\) and \(y\) coordinates.

\[
\text{Midpoint} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
\]

Example: Find the midpoint and distance between \((4,3)\) and \((-2, 5)\).

Practice: Find the midpoint and distance for each pair of points. Leave the distances in radical form.

1. \((8, -2)\) and \((4, -5)\)  
2. \((-3, -6)\) and \((7, 3)\)  
3. \((8, 9)\) and \((3, 1)\)

Parallel lines have the same slope.  
Perpendicular lines have negative reciprocal slopes.

Example: Which lines are parallel? Which are perpendicular?

a. \(2x - 3y = 7\)  
b. \(2x + 3y = 7\)  
c. \(3x - 2y = 7\)  
d. \(y = \frac{2}{3}x - 7\)

Example: The points \(A(0,7)\) \(B(5,8)\) \(C(8,2)\) and \(D(3,1)\) form a quadrilateral. Is quadrilateral \(ABCD\) a parallelogram?

Practice:
1. What is the equation of a line perpendicular to \(5x - y = 7\) through the point \((9,1)\) in Standard Form?

2. Points \(A(4,3)\) \(B(-8,1)\) and \(C(5,-3)\) are graphed on the plane to form a right triangle. Which vertex is the right angle of triangle \(ABC\)?

3. What is the approximate perimeter of triangle \(ABC\) above (to the tenth)?
If an equation is linear, it has a constant slope. Many EOC problems will ask you to find an answer using a linear model.

Examples:
In 1970, the average life expectancy in the U.S was 75.2 years. In 2000, the average life expectancy was 78.8 years. Assuming the trend is linear, what will be the average life expectancy in 2020?

The oak tree in your backyard is 15 feet tall. When you planted it 2 years ago, it was just 7 feet tall. If the growth can be modeled by a linear equation, how tall will the tree be in 5 years?

Practice
Assume all growth or depreciation is linear for the following:

1. In the year 2000, a 40-inch LCD television cost about $2,500. In 2008, you can buy the same television for about $1,100. Assuming a linear rate of depreciation, how much less does the television cost this year than last year?

2. Attendance at Lincoln high school has increased linearly for the past 10 years. 5 years ago, Lincoln had 1,235 students. Now Lincoln has 1,705 students. If the growth continues, how many students will attend Lincoln in 3 years?

3. The height of a burning candle can be expressed by a linear equation where \( h \) is the height and \( m \) is the number of minutes the candle has been burning. If a 15-inch candle burns for 35 minutes, it will be 8 inches tall. What equation can be written for the height \( h \) of a 15-inch candle that has been burning for \( m \) minutes?

Practice: Write a linear equation in slope-intercept form for each:

1. A restaurant takes 1 hour to prepare and bake 16 pizzas. To prepare and bake 20 pizzas takes the same restaurant takes 76 minutes. What equation can be used to represent the time in minutes \( m \) to bake \( p \) pizzas?

2. The length of a particular snake is 16 inches at age 1 and 30 inches at age 3. Assuming the snake grows at a linear rate, how long was it at birth?
Exponential Growth/Depreciation

**Exponential growth:** \( V = p(1+r)^t \)

Amount  principal  rate (percent as a decimal)  time (usually in years)

**Examples:**
How much will $400 be worth in 5 years at 7% interest (to the cent)?

How long will it take for your money to double earning 6% interest?
   A. 1.5 years  B. 10 Years  C. 12 Years  D. 120 years

**Practice:**
1. The value of an automobile depreciates exponentially. If the rate of depreciation on your 7 year-old car is 20%, and the purchase price was $16,000, approximately how much is the car worth today?
   A. $18,380  B. $13,890  C. $3,748  D. $3,355

2. Approximately what interest rate must you earn on an investment to double the value of your money every 15 years?
   A. 4%  B. 5%  C. 6%  D. 7%

3. In 2001, the population of Poughkeepsie was 12,500. In 2007, the population had increased by 1,170. Assuming that the growth rate is exponential, what is the annual rate of growth?
   A. 1.2%  B. 1.5%  C. 9.4%  D. 15.0%

**Practice:**
1. How long will it take an investment to triple in value if it is earning 4.2% interest annually?
   A. 17 years  B. 20 years  C. 23 years  D. 26 years

2. Allentown has a population of 50,000 and a 1.5% annual population growth rate. Brighton has 40,000 and a growth rate of 1.8%. If these growth rates continue, approximately how many years will it take for the populations to be equal?
   A. 25 years  B. 50 years  C. 75 years  D. 100 years

3. An aggressive species of vine grows in length by 20% daily. If the vine is 8 inches long when planted, about how long will it be after two weeks of growth?
   A. 4 feet  B. 5.5 feet  C. 7 feet  D. 8.5 feet
The vertex: \[ x = \frac{-b}{2a} \] To find the y-coordinate, plug-in x.

The Roots (solutions, zeros) \[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Examples:
Find the roots and vertex for the following quadratic equation:
\[ y = 4x^2 + 4x - 3 \]

Solve for x: \[ 3x^2 - 13x = 10 \]

A. \[ \left\{ \frac{2}{3}, 5 \right\} \]  
B. \[ \left\{ -\frac{2}{3}, 5 \right\} \]  
C. \[ \left\{ \frac{2}{3}, -5 \right\} \]  
D. \[ \left\{ -\frac{2}{3}, -5 \right\} \]

Practice:
1. The formula for the height of a ball h after t seconds is given by the formula: \[ h = -16t^2 + 64t + 4 \]
   What is the maximum height of the ball?
   A. 4 feet  
   B. 60 feet  
   C. 64 feet  
   D. 68 feet

2. Find the roots: \[ h = 4x^2 + 16x + 15 \]
   A. \[ \left\{ \frac{3}{2}, \frac{5}{2} \right\} \]  
   B. \[ \left\{ -\frac{3}{2}, \frac{5}{2} \right\} \]  
   C. \[ \left\{ \frac{3}{2}, -\frac{5}{2} \right\} \]  
   D. \[ \left\{ -\frac{3}{2}, -\frac{5}{2} \right\} \]

3. The length of a rectangle is 4 inches greater than twice the length. If the rectangle has an area of 70in^2, what is its perimeter?
   A. 34in  
   B. 38in  
   C. 40in  
   D. 74in

Quadratics
Systems of Equations

Examples:

Where would the graphs of the two equations below intersect?
A. \( y = 4x - 3 \)  
B. \( 3x - 2y = -9 \)

Write and solve a system of equations to solve the following:
At a local bakery, 3 pastries and 5 doughnuts cost $6.75.
At the same bakery, 4 pastries and one doughnut cost $4.75.
How much would 2 pastries and 2 doughnuts cost?
A. $3.25  
B. $3.50  
C. $3.75  
D. $4.00

Practice:

1. Solve for \( x \) in the following system of equations:
\[ 2x - y = 4 \]
\[ 3x - 5y = 8 \]
A. \( x = \frac{7}{12} \)  
B. \( x = \frac{12}{7} \)  
C. \( x = \frac{28}{13} \)  
D. \( x = \frac{13}{28} \)

2. Micah has 4 more pencils than he has erasers. If he has a total of 42 pencils and erasers, how many pencils does he have?
A. 19  
B. 21  
C. 23  
D. 25

Practice:

1. The perimeter of a rectangle is 31 inches, and the width is 10 inches less than twice the length. What is the width of the rectangle?
A. 7in  
B. 7.5in  
C. 8in  
D. 8.5in

2. Adult tickets to a theme park cost $19 and children’s tickets cost $15. One cashier collected $632 from 40 park attendants. How many adult attendants paid the cashier?
A. 8  
B. 16  
C. 24  
D. 32
Formulas you must know:

Fill-in the following formulas. These are all formulas that you must be able to use on the EOC test Tuesday. You will be quizzed on these formulas Wednesday so memorize these!

1. Point-Slope Form: _____________________________________________

2. Slope-Intercept Form: ___________________________________________

3. Standard Form: _________________________________________________

4. Slope (given two points): _________________________________________

5. Distance Formula (given two points): ________________________________

6. Midpoint Formula (given two points): ________________________________

7. Quadratic Formula: ______________________________________________

8. X-coordinate of the vertex of a quadratic: ____________________________

9. Slope of a standard form linear equation: ____________________________

10. Direct Variation: ________________________________________________

11. Exponential Growth: _____________________________________________

Go to

http://www.ncpublicschools.org/accountability/testing/eoc/sampleitems/alg1scs2003extset

for sample items if you are looking for something to review tonight but GET A GOOD NIGHT’S REST!