Quadratic Equations and Functions:

$$
y=2 x^{2}-7 \quad y=5 x^{2}-5 x-1 \quad f(x)=(x-3)^{2}-6
$$

Once again, we will begin by graphing quadratics using a table of values.
Examples: Graph each using the domain given and a table of values.

1. $y=x^{2}$
\{D: -3, -1, 0, 1, 3\}
2. $y=x^{2}-2$
\{D: -3, -1, 0, 1, 3\}
3. $y=(x-3)^{2}$
\{D: 0, 2, 3, 4, 6\}

## Practice:

Graph the following by using your own values for the domain.
Plot at least 5 points on each graph. It should look like a parabola.

1. $y=2 x^{2}$
2. $y=\frac{1}{2} x^{2}-3$
3. $y=(x+3)^{2}+1$



## Recognizing graphs:

There are several things you should recognize about the graphs of quadratics.

## 1. Right-Side-Up/ Up-Side-Down

Graph each using the domain:
\{D: -3, -1, 0, 1, 3\}
a. $y=2 x^{2}-9$
b. $y=-2 x^{2}+9$
notes:


When $\mathbf{A}$ is negative, the typical parabola is flipped up-side-down.

## Roots and vertex:

Roots are the x -intercepts.
The vertex is the tip of the ' $U$ '.

## Practice:

Graph and name the roots and vertex for each:
a. $y=x^{2}-4 x+3$
\{D: -1, 0, 1, 2, 3, 4, 5\}
b. $y=x^{2}+4 x$
\{D: -5, -4, -3, -1, 0, 1\}


Graph each and fill-in the blanks for the questions that follow.

1. $y=(x-3)^{2}-9$
\{D: 0, 1, 2, 3, 4, 5, 6 \}
Upside-Down or
Right-Side-Up? $\qquad$
Vertex: $\qquad$
Roots: $\qquad$

2. $y=-x^{2}-6 x-5$
\{D: -6, -5, -4, -3, -2, -1, 0\}
Upside-Down or
Right-Side-Up? $\qquad$

y-intercept: $\qquad$

## Graph each and fill-in the blanks for the questions that follow.

3. $y=-x^{2}+4$
\{D: -3, -2, -1, 0, 1, 2, 3\}
Upside-Down or
Right-Side-Up?
Vertex: $\qquad$
Roots: $\qquad$

4. $y=3 x^{2}+12 x+7$
\{D: -4, -3, -2, -1, 0\}
Upside-Down or
Right-Side-Up? $\qquad$


## Vertex Form:

Graph the following equations using a table of values.

1. $y=(x-7)^{2}-6$
\{D: 3, 5, 7, 9, 11\}
Vertex: $\qquad$
2. $y=(x+1)^{2}+3$
$\{D:-3,-2,-1,0,1\}$
Vertex: $\qquad$


There are several things we can recognize from an equation written in Vertex Form. (note: Vertex form is actually generally called standard form, but that is confusing... so I call it vertex form).

## For an equation:

$$
y=\left(x-x_{1}\right)^{2}+y_{1} \text {, the vertex is at }\left(x_{1}, y_{1}\right)
$$

Example: Name the vertex of each.

$$
y=(x+4)^{2}-9 \quad y=(x-3)^{2}+5
$$

Practice: Name the vertex of each equation.

1. $y=(x-4)^{2}-8$
2. $y=(x-2)^{2}+1$
3. $y=3(x+1)^{2}-3$
4. $y=x^{2}-6 x+5$

C

## Vertex Form:

$y=(x-2)^{2}+6$ is equal to $y=x^{2}-4 x+10$

Converting $y=x^{2}-4 x+10$ to $y=(x-2)^{2}+6$ is called

## Completing the square:

$$
y=x^{2}-4 x+2
$$

1. Forget about the +2 (for now).
2. $(x-2)^{2}=x^{2}-4 x+4$ so $(x-2)^{2}-4=x^{2}-4 x$
3. Bring Back the +2 .
4. Done.

## More Examples:

Complete the square to put each equation into vertex form. Name the vertex.

1. $y=x^{2}-6 x+3$
2. $y=x^{2}-10 x+24$

## Practice:

Complete the square to put each equation into vertex form. Name the vertex.

1. $y=x^{2}-8 x+11$
2. $y=x^{2}+2 x-5$
3. $y=x^{2}-8 x-1$
4. $y=x^{2}-3 x+5$

## Practice:

1. $y=x^{2}-x+1$
2. $y=2 x^{2}-8 x-14$

Vertex Form: Convert each into Vertex Form. State the vertex for each problem.

1. $y=x^{2}-6 x+7$
2. $y=x^{2}-18 x+80$
3. $y=x^{2}-3 x+1$
4. $y=x^{2}-14 x+50$
5. $y=x^{2}+x-3$
6. $y=x^{2}-11 x+2$
7. $y=x^{2}+6 x$
8. $y=x^{2}+13 x$

Challenge:
9. $y=(x+3)(x-6)$
10. $y=2 x^{2}+6 x+14$

## Graph the following by:

1. Convert to vertex form if necessary.
2. Create a table of values.
3. Graph and connect.
4. $y=(x-2)^{2}-5$
5. $y=x^{2}-6 x+2$

6. $y=2(x-3)^{2}-4$
7. $y=x^{2}+8 x+11$


## Review:

Completing the square:

$$
y=x^{2}-2 x-15
$$

$$
y=(x-1)^{2}-16
$$

Finding the Roots is easy.
The roots are the $x$-intercepts, which occur where $y=0$.
Set y to 0 and solve.
$0=(x-1)^{2}-16$

## Remember:

Every positive number has two square roots. Solve for both.

## More Examples:

Complete the square then solve for the roots.
State the Vertex and Roots.

1. $y=x^{2}-6 x+8$
2. $y=x^{2}+10 x-24$

## Practice:

Complete the square then solve for the roots. State the Vertex and Roots.

1. $y=x^{2}-8 x-9$
2. $y=x^{2}+3 x-10$

## Vertex Form: Convert each into Vertex Form, then solve for the roots.

 State the vertex and roots for each problem. Check your roots by factoring.1. $y=x^{2}-4 x+3$
v: $\qquad$ r: $\qquad$
2. $y=x^{2}-6 x+8$
v:
r:

3. $y=x^{2}-x-12$
4. $y=x^{2}-14 x+45$
$\qquad$ r: $\qquad$
5. $y=x^{2}+8 x+15$

r:
6. $y=x^{2}-5 x-14$
7. $y=x^{2}-3 x-28$
v: $\qquad$ r: $\qquad$
v:

r: $\qquad$
8. $y=x^{2}-2 x$

All of the problems we have done so far could have been done by factoring (which we learned in the last unit).

1. $y=x^{2}-8 x-9$
2. $y=x^{2}+3 x-10$

## When the Roots are not Rational:

Example: Find the roots.

$$
y=(x-5)^{2}-3
$$

## Sometimes there are NO ROOTS:

Example: Find the roots.

$$
y=(x-5)^{2}+3
$$

## Practice:

Complete the square then solve for the roots.
State the Vertex and Roots.

1. $y=x^{2}-6 x-5$
2. $y=x^{2}+7 x+13$

Challenge: (these only look hard)
Complete the square then solve for the roots.
State the Vertex and Roots.

1. $y=x^{2}-4 x+3.75$
2. $y=x^{2}-7 x-12.25$

Vertex Form and Roots: Convert each into Vertex Form.
State the vertex and the roots for each problem.
Round decimal roots to the tenth.

1. $y=x^{2}-12 x+35$
2. $y=x^{2}+4 x-12$
3. $y=x^{2}+2 x-35$
4. $y=x^{2}+14 x+49$
5. $y=x^{2}-11 x+30$
6. $y=x^{2}-x-20$
7. $y=x^{2}-3 x+4$
8. $y=x^{2}-4 x+6$
9. $y=x^{2}+5 x-3.75$
10. $y=x^{2}-x+1$

## Graph the following by:

1. Convert to vertex form if necessary.
2. Find the roots. Round decimal roots to the tenth.
3. Create a table of values.
4. Graph and connect.
5. DO YOUR ROOTS MAKE SENSE?
6. $y=x^{2}-2 x-5$

Vertex: $\qquad$
Roots: $\qquad$ and $\qquad$
12. $y=x^{2}+7 x+9$

Vertex: $\qquad$
Roots: $\qquad$ and __-_-_


In most of the equations we have graphed so far had $a=1$.
Completing the Square can be more difficult when $a$ is not 1 , but it is still possible (but not recommended):

Ex. $y=2 x^{2}-7 x+9$

There is a less complicated way!
The axis of symmetry is the vertical line which passes through the vertex of a quadratic. The x-coordinate of the vertex (also called the axis of symmetry) can be found by:

$$
x=\frac{-b}{2 a}
$$

## Deriving the axis of symmetry:

$$
y=a x^{2}+b x+c
$$

Finding the vertex using the axis of symmetry:

$$
\text { Ex. } y=-2 x^{2}+12 x-19 \quad y=3 x^{2}-9 x+10
$$

## Practice:

Use the axis of symmetry to find the Vertex.

1. $y=4 x^{2}-6 x-5$
2. $y=2 x^{2}-9 x+1$

## Practice:

Use the axis of symmetry to find the Vertex (to the hundredth).

1. $y=10 x^{2}-2 x-3$
2. $y=3 x^{2}-x+2$
3. $y=\frac{1}{3} x^{2}-x+4$
4. $y=6 x^{2}-5 x+2$

Find the roots and vertex of each. Round decimal roots to the tenth. DO THESE BY COMPLETI NG THE SQUARE, YOU MAY ALSO FACTOR.

1. $y=x^{2}+2 x-24$
2. $y=x^{2}-13 x+42$
vertex: $\qquad$ roots: $\qquad$
3. $y=x^{2}+12 x+35$
vertex: $\qquad$ roots: $\qquad$
4. $y=x^{2}+2 x-4$
vertex: $\qquad$ roots: $\qquad$ vertex: $\qquad$ roots: $\qquad$

Find the vertex of each. Round decimal roots to the tenth. DO THESE USI NG THE AXIS OF SYMMETRY.
5. $y=2 x^{2}-x-3$
6. $y=8 x^{2}-4 x+1$
vertex:

7. $y=5 x^{2}-8 x-17$
8. $y=6 x^{2}-8 x-3$

Find the roots and vertex of each. Round decimal roots to the tenth. DO THESE BY COMPLETI NG THE SQUARE OR USI NG $-b / 2 a$.

1. $y=x^{2}-6 x+5$
2. $y=x^{2}+2 x-35$
vertex:
3. $y=4 x^{2}+10 x-1$
vertex:
4. $y=3 x^{2}+12 x-3$
$\qquad$
5. $y=x^{2}-10 x+25$
6. $y=4 x^{2}-5 x+11$
vertex:
7. $y=10 x^{2}-3 x-4$
8. $y=5 x^{2}-35 x+60$

## Try to solve the following questions involving polynomials:

9. For $(x-2)(x+b)=x^{2}+5 x+c$, what are the values of $b$ and $c$ ?
10. The equation $y=x^{2}-6 x+c$ has only one root. What is $c$ ?

$$
c=
$$

11. If $x^{2}-8 x+c=(x-b)^{2}-1$, what are the values of $b$ and $c$ ?
b = __-_-_-_

$$
\mathrm{c}=\text { _-------- }
$$

12. If $(x+3)(x-b)=(x-2)^{2}-c$, what are the values of $b$ and $c$ ?

$$
\begin{aligned}
& \mathrm{b}= \\
& \mathrm{c}=
\end{aligned}
$$

## GRAPHS:

Find all of the coordinate pairs that will fit on a standard graph for the following equation: 100 pts. each. $y=x^{2}+2 x-8$

ROOTS:
100. $y=(x-3)(x+7)$
200. $y=x^{2}+2 x-3$
300. $y=x^{2}-5 x+3.25$
400. $y=x^{2}-4$

VERTEX:
100. $y=(x-3)^{2}-1$
200. $y=x^{2}+2 x-4$
300. $y=2 x^{2}-3 x+8$
400. $y=2(x-3)(x+1)$

ROOTS:
100. $y=x^{2}-6 x+5$
200. $y=x^{2}+x-2$
300. $y=2 x^{2}-6 x+4$
400. $y=9 x^{2}-49$

VERTEX:
100. $y=x^{2}-4 x+19$
200. $y=3 x^{2}+6 x-1$
300. $y=11 x^{2}-3 x+19$
400. $y=\frac{x^{3}+3 x^{2}+2 x}{x+2}$

Graph each equation below. Plot at least five points for each.

1. $y=x^{2}-6 x+2$
2. $y=x^{2}-7 x+8$


State the VERTEX for each of the equations below.
3. $y=x^{2}-6 x+2$
3.
4. $y=x^{2}-5 x+9$
4.
5. $y=4 x^{2}-3 x+1$
5.
6. $y=3 x^{2}-7 x+4$
6.

State the ROOTS for each of the equations below.
Write NO ROOTS if there are none.
Write the answer in radical form, then state both roots. Ex. $\quad x= \pm \sqrt{4}-1 \quad \underline{1} \boldsymbol{\&} \underline{\mathbf{- 3}}$
7. $y=(x-3)^{2}-25$
7. $\qquad$
$\qquad$
8. $y=x^{2}-12 x+29$
8. $\qquad$ \& $\qquad$
9. $y=x^{2}+6 x+10$
9. $\qquad$
10. $y=x^{2}-7 x+9$
10. $\qquad$

Find the roots and vertex of each by completing the square．
Round decimal roots to the tenth．
1．$y=x^{2}-4 x+3$
2．$y=x^{2}+6 x-4$
vertex： $\qquad$ roots： $\qquad$ vertex： $\qquad$ roots： $\qquad$

Find the vertex using the axis of symmetry（ $x=-b / 2 a$ ）

3．$y=x^{2}-4 x+3$

Name $\qquad$ Period
cose
Find the roots and vertex of each by completing the square．
Round decimal roots to the tenth．
1．$y=x^{2}-4 x+3$
2．$y=x^{2}+6 x-4$
vertex： $\qquad$ roots： $\qquad$ vertex： $\qquad$ roots： $\qquad$

Find the vertex using the axis of symmetry（ $x=-b / 2 a$ ）

3．$y=x^{2}-4 x+3$
vertex：

We have learned to find the roots of a Quadratic by completing the square, or by factoring:

Ex. $y=x^{2}-10 x+24$

If you cannot easily factor or complete the square:
The Quadratic Formula can be used to find the roots of any quadratic equation. It is long, and sometimes difficult to memorize.

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

## Deriving the Quadratic Formula:

Set $y=0$ and solve for $x$ by completing the square.

$$
y=a x^{2}+b x+c
$$

Finding the roots using the Quadratic Formula:

$$
\text { Ex. } y=6 x^{2}-11 x+3 \quad y=2 x^{2}-7 x+1
$$

## Practice:

Find the roots using the Quadratic Formula:

1. $y=x^{2}-3 x-10$
2. $y=3 x^{2}-9 x+4$

Special Cases: One Root or No Roots Find the roots using the Quadratic Formula:

1. $y=25 x^{2}-10 x+1$
2. $y=3 x^{2}-4 x+2$

Roots are 'zeros' of a Quadratic.
Finding the roots is the same as solving for $\mathrm{y}=0$.
To solve a quadratic like the one below, get zero on one side of the equation and then use the Quadratic Formula.

Ex. $9 x=2 x^{2}+9$

Practice: Solve for x :

1. $2 x^{2}-7 x+5=0$
2. $10=x^{2}-3 x$

Practice: Solve for x :

1. $x^{2}+7=5 x$
2. $7=2 x^{2}-5 x$

Challenge: Solve using a quadratic equation.
The width of a sheet of cardboard is three inches more than twice the length. If the area is $77 \mathrm{in}^{2}$, what are the length and width of the cardboard rectangle?

Solve each of the quadratic equations below for $x$ and place the solutions in order to answer the question at the bottom of the page. Some letters may be used more than once. If there are two solutions, list two letters. If there are no solutions, place an I.

1. $0=2 x^{2}-7 x+3$
2. $0=5 x^{2}+8 x-21$
3. $0=2 x^{2}+7 x+9$
4. $10=7 x^{2}+9 x$
5. $18 x=x^{2}+81$
6. $4 x=x^{2}+5$
7. $9 x^{2}+42 x=-49$
8. $x=5 x^{2}+7$
9. $4 x+4=3 x^{2}$
10. $2-x=6 x^{2}$
$\mathbf{A}=2 \quad \mathbf{M}=-7 / 3 \quad \mathbf{H}=3 \quad \mathbf{E}=7 / 5 \quad \mathbf{D}=-3 \quad \mathbf{S}=5 / 7 \quad \mathbf{C}=-2 \quad \mathbf{T}=1 / 2 \quad \mathbf{R}=9 \quad \mathbf{N}=-2 / 3 \quad \mathbf{I}=$ no solutions
What is the name given to the part of the Quadratic Formula found under the square root?

## Word problems that involve quadratics:

Sometimes solving a system of equation involves solving a quadratic equation.

## Ex.

John is seven years older than his brother. The product of their ages is 120. How old is John?

## Quadratics can be used to find non-integer solutions as well:

## Ex.

A rectangular rug is three feet longer than it is wide. The area of the rug is $80 \mathrm{ft}^{2}$. What are the dimensions of the rug?

Practice: Solve.

1. Callie is five years more than three times as old as her sister Tanya. The product of their ages is 25 . How old is Callie?
2. The windows on a house are two feet taller than they are wide. The area of each window is $14 \mathrm{ft}^{2}$. How many feet tall are the windows? How many inches is this?

## Sum/ Difference and Product problems:

Ex.
The sum of two numbers is 15 and their product is 50 . Find the numbers.
Ex.
The product of two numbers is 8 and their difference is 1 . Find their sum.

Practice: Solve. Round to the hundredth.

1. The sum of two numbers is 7 . Their product is 11 . Find their difference.
2. The difference of two numbers is 3 , and their product is 60 . Find their sum.

## Challenge:

The perimeter of a rectangle is 28 cm and its area is $40 \mathrm{~cm}^{2}$. What are the lengths of its sides?
$\qquad$

Solve each: Round decimal answers to the hundredth.

1. Alexis and Maria each choose a number between 1 and 20 . Kevin looks at the numbers and tells them that Maria's number is one more than twice Alexis' number. Bohdan multiplies the numbers and gets 36 .

What was Maria's number?

1. $\qquad$
2. The area of a rectangle is $60 \mathrm{~cm}^{2}$. The width of the rectangle is 2 cm more than the height.

Find the perimeter of the rectangle.
2. $\qquad$
3. A rectangular baking dish is 3 inches longer than it is wide. The area of the base of the dish is 108 square inches.

What is the width of the dish?
3. $\qquad$
4. Callie is five years more than three times as old as her sister Tanya. The product of their ages is 25 . How old is Callie?

Solve each: Round decimal answers to the hundredth.
4. The difference of two numbers is 9 and their product is 100 .

What is the smaller of the two numbers?
4. $\qquad$
5. The sum of two numbers is 16 and their product is 30 .

What is the difference of the two numbers?
5. $\qquad$

Challenge. The Pythagorean Theorem should be familiar to you. In any right triangle, the sum of the squares of the legs equals the square of the hypotenuse. $a^{2}+b^{2}=c^{2}$


What is the perimeter of the triangle below?

6.

Convert to vertex form:
100. $y=x^{2}-4 x+1$
200. $y=x^{2}-7 x-2$

State the vertex for each:
Round decimal answers to the hundredth.
100. $y=(x-3)^{2}+4$
200. $y=x^{2}-3 x-1$
300. $y=3 x^{2}-6 x+5$
400. $y=-3 x^{2}-10 x+6$

State the solutions for each:
Round decimal answers to the hundredth.
100. $0=(x-3)^{2}-4$
200. $0=2 x^{2}-9 x-5$
300. $7 x=5 x^{2}+3$
400. $4=5 x^{2}-2 x$

Solve each:
Round decimal answers to the hundredth.
400. The product of two numbers is 15 and their sum is 8 . What is their difference?
500. The area of a rectangle is $54 \mathrm{~cm}^{2}$. The length is seven centimeters more than the width. What is the perimeter of the rectangle?
600. The area and perimeter of a rectangle is 20 . What is the length of the shorter side?

Graph the equation below. Plot at least five points.

1. $y=x^{2}+12 x+33$


State the vertex for each equation below.
2. $y=x^{2}-4 x+11$
2.
3. $y=x^{2}-8 x+3$
3.
4. $y=x^{2}-5 x-3$
5. $y=2 x^{2}-16 x+12$
5. $\qquad$
6. $y=5 x^{2}-2 x+1$
6.

Find the solution(s) for each equation below. Write 'no solution' where applicable.
7. $0=x^{2}+4 x-21$
7.
8.
9. $3=3 x^{2}+2 x$
10. $0=5 x^{2}-9 x$
10.

Solve each. Round decimal answers to the hundredth.
11. James is five years older than Jack. The product of their ages is 36 . How old is James?
12. The sum of two numbers is 7 . Their product is 12 . What is the smaller of the two numbers?
13. The area of a rectangle is $34 \mathrm{~cm}^{2}$. If the length is 7 less than 5 times the width, what is the perimeter of the rectangle?
14. When I multiply two numbers I get 42. If I subtract the smaller number from the larger one, I get 2. What will I get if I add the numbers?
11. $\qquad$
12.
13. $\qquad$
14.

## Graphing Quadratics using the TI-83 calculator:

## $\mathbf{Y}=$

This is where you enter equations to be graphed on your calculator.
Begin by entering the following equations, then hit GRAPH:
$\backslash \mathbf{Y}_{1}=-2 x^{2}-6 x+1$
$1 \mathbf{Y}_{2}=(x-3)^{2}-6$

## ZOOM

Try graphing the following equation: $y=2 x^{2}-30 x+45$
Erase the previous equations from $Y=$

## What do you notice?

Use the ZOOM functions of your graphing calculator to zoom out until you can see the graph of the parabola. Use the ZOOM Box (ZBox) to box the parts of the graph you are interested in.

## WINDOW

In addition to using the ZOOM function, you can us the WI NDOW function to change what you see of the graph.
Try graphing the following equation: $y=x^{2}-50 x+60$
Erase the previous equations from $Y=$
None of the ZOOM functions make the graph easy to see, so it helps to list what we know about the graph. Convert the equation into vertex form: $y=(x-25)^{2}-565$

We know the vertex: $\qquad$
and the $y$-intercept: $\qquad$ (look at the original equation and set $x=0$ )

## Hit WI NDOW.

Choose values for Xmin, Xmax, Ymin, and Ymax which will help you see the graph.
Hint: The Ymin should be below -565. The Ymax should be above 60. Why?

Finding the ROOTS and VERTEX is done using the CALC function． CALC is found above the TRACE button below the screen．

## Begin by entering the following equations，then hit GRAPH：

$\backslash \mathbf{Y}_{1}=x^{2}-6 x+1$
$\backslash \mathbf{Y}_{\mathbf{2}}=x^{2}+10 x+18$

## Get to the CALC menu（2nd TRACE）：

We will begin by calculating the VERTEX．Choose MI NI MUM（the vertex of both parabolas above is a minimum，for an upside－down parabola the vertex is a maximum）．
The questions below appear on the screen：
Left Bound？Choose a point left of the vertex．（Hit Enter）
Right Boud？Choose a point right of the vertex．（Hit Enter）
Guess？Choose a point near the vertex．（Hit Enter）
The calculator can be off by a little．If the calculator says 7.9999989 for a coordinate，it is likely 8 ．What is the vertex of each equation above？（use only the calculator，you may check your answer with another method）
$\backslash Y_{1}=x^{2}-6 x+1$ v：＿＿＿－＿－＿－＿
$\backslash \mathbf{Y}_{\mathbf{2}}=x^{2}+10 x+18 \mathbf{v}:$
$\qquad$

We will now calculate the ROOTS．Choose ZERO from the CALC menu．
The zeros are the $x$－intercepts，also called roots．
Answer the questions again．
What are the roots of each equation above？

$$
\backslash \mathbf{Y}_{1}=x^{2}-6 x+1 \mathbf{r}: \ldots,-\ldots \quad \backslash \mathbf{Y}_{2}=x^{2}+10 x+18 \mathbf{r}:
$$

$\qquad$ ，＿－＿－＿

Homework：Find the roots and vertex of each．Round to the tenth or use bar notation．
1．$y=3 x^{2}-4 x-7$

v： $\qquad$
2．$y=-2 x^{2}+17 x-36$
r：

v： $\qquad$
3．$y=0.01 x^{2}-1.3 x-10.5$
r：

v： $\qquad$

Vertical Motion can be described using a Quadratic Equation．

$$
h=-16 t^{2}+v t+c(\text { where } \mathbf{h} \text { is in feet, } \mathbf{v} \text { is in feet/ sec. })
$$

$h=h e i g h t$ in feet
$t=$ time
$\mathrm{v}=$ initial velocity
$\mathrm{c}=$ initial height
points will be given as（ $t, h$ ）
In this equation，-16 represents the force of gravity $\left(-16 \mathrm{ft} / \mathrm{sec}^{2}\right)$ ．
In Metric units（meters）we would use -4.9 meters $/ \mathrm{sec}^{2}$ ．

$$
h=-4.9 t^{2}+v t+c \quad(\text { metric units })
$$

## Practice：

1．Chase stands on his garage and kicks a soccer ball with an upward velocity of 60 feet per second．When the ball leaves his foot，it is $\mathbf{2 5}$ feet above his driveway．
a．Write an equation for this situation．
a．
b．How long does it take the ball to reach its maximum height？
b．
c．How high does the ball get？
c．
d．How long does it take the ball to land on the driveway？
d．
e．If Chase falls off the garage，how long will it take him to break his ankle on the pavement below？（Do not attempt this at home．）
e．
$\qquad$
2. Tiger Woods hits a golf ball with an initial UPWARD velocity of 29.4 meters per second (we are not calculating distance, only height and time). The golf ball lands on the green, which is 12 meters below the tee box (use this for the initial height, the tee box is $\mathbf{1 2}$ meters ABOVE the green).
a. Write an equation for this situation.
a. $\qquad$
b. How long does it take the ball to reach its maximum height above the green?
b.
c. How high does the ball get?
C.
d. How long does it take the ball to land on the green?
d.
e. How high above the green is the ball after 4 seconds?
e.
f. How high above the green is the ball after 6 seconds?
f.
3. Mike fires his rifle with an initial velocity of 1,152 feet per second straight into the air while standing on the rim of the Grand Canyon, which is about 5,000 feet deep. (Use 5,000 ft as the initial height)
a. Write an equation for this situation.
a.
b. How long does it take the bullet to reach its maximum height above the Colorado River?
b.
.__-_-_-
c. How high does the bullet get?
C.
d. How long does it take the bullet to land in the river?
e. How high above the rim is the bullet after one minute?
e.
f. How long will it take the bullet to pass by you (on the way down) at the height of the canyon rim?

Review: Vertical Motion can be described using a Quadratic Equation.
$h=-16 t^{2}+v t+c$ (where $h$ is in feet, $v$ is in feet/ sec.)
$h=$ height in feet
$t=$ time
$\mathrm{v}=$ initial velocity
$\mathrm{c}=$ initial height
points will be given as ( t , h )
In this equation, -16 represents the force of gravity $\left(-16 \mathrm{ft} / \mathrm{sec}^{2}\right)$.
In Metric units (meters) we would use -4.9 meters $/ \mathrm{sec}^{2}$.

$$
h=-4.9 t^{2}+v t+c \quad \text { (metric units) }
$$

Practice: Round decimal answers to the tenth.

1. Jamie is using her slingshot to try and hit birds in a nearby tree (she is not a very nice person). She fires small pebbles with an initial upward velocity of 45 feet per second. When she fires the pebbles, she holds the slingshot 5 feet above the ground.
a. Write an equation for this situation.
a. $\qquad$
b. How high can she fire a pebble?
b.
c. A pebble hits a bird 2 seconds after she shoots it. How high is the bird?
c. $\qquad$
d. Does the pebble in part c hit the bird on the way up or down?
d.
.__-_-_-
e. How long will each pebble stay in the air?
e.

2. Ashley fires a small rocket in her backyard with an initial upward velocity of 78.4 meters per second. The rocket launches from ground level.
a. Write an equation for this situation.
a.
b. How high will the rocket be after 4 seconds?
b.
c. How long does it take the rocket to reach its maximum height?
c.
d. How high will the rocket get?
d.
e. How long will it take to land (without a parachute)?
e.
f. How long will it take the rocket to reach 100 feet?

For this problem: set $\mathrm{h}=100$, then find the roots. There will be two times when the rocket is at 100 feet. Try to find both.

## f.

3. Kevin drops (zero initial upward velocity) a penny from the tallest building in the world, Taipei 101 in Taiwan. The tower is $\mathbf{1 , 6 7 0}$ feet tall.
a. Write an equation for this situation.
a. $\qquad$
b. How long does it take the penny to hit the ground?
b._-_-_-_
c. How high is the penny after:

3 seconds $\qquad$
6 seconds $\qquad$
9 seconds $\qquad$
12 seconds $\qquad$
4. Challenge: How much upward initial velocity will J oe need to throw a basketball over his house if his house is 32 feet tall? Explain your answer. You can assume that the initial height is zero.
$\qquad$

Practice: Round answers to the hundredth.

1. Robbie Knievel is attempting another one of his hairbrained stunts, driving his motorcycle up a ramp and attempting to launch over a three-story building. If he rides $\mathbf{7 0 m p h}$ up a $\mathbf{4 5}$-degree ramp, he can achieve an upward velocity of $\mathbf{7 0}$ feet per second. The top of the ramp is $\mathbf{1 4}$ feet high.
a. Write an equation for this situation.

## a.

$\qquad$
b. How long will it take Robbie to reach his maximum height?
b.
c. How high will Robbie jump?
c.
d. How long will Robbie spend in the air before he lands (assume he lands at ground level, even though he would probably have a ramp of some sort).
d.
e. If the ramp is 20 feet high, he can only achieve an upward velocity of 65 feet per second. How high will he get if he increases the height of his ramp to 20 feet?

## e.

2. At a wedding, a champagne bottle is popped, launching the cork with an upward velocity of 25 meters per second. The bottle was being held 1.5m above the floor when the cork popped off.
a. Write an equation for this situation.
a. $\qquad$
b. How high is the cork after: $\mathbf{1}$ sec $\qquad$ 2 sec $\qquad$ 3 sec $\qquad$ 4 sec $\qquad$
c. How high does the cork get?
C.
C.
d. How long does the cork take to land?
d.
._-_-_-_

Review
Find the vertex and roots:
3. $y=-2 x^{2}+7 x+27$
4. $y=3 x^{2}-8 x-5$

Vertex: $\qquad$

Roots: $\qquad$
5. $h=-16 t^{2}+45 t+80$

Vertex: $\qquad$

Roots:
\& $\qquad$

Vertex: $\qquad$

Roots: $\qquad$ \& $\qquad$
7. $y=3 x^{2}-27$
8. $y=(x-7)(x+3)$

Vertex: $\qquad$

Roots: $\qquad$ \&

## Solve each. Round decimal answers to the hundredth.

1. Parker is half a year older than twice his sister Julie's age. The product of their ages is 34. How old is Parker?
2. 
3. The perimeter of a rectangle is 21 cm , and the area $27 \mathrm{~cm}^{2}$. What is the length of the longer sides of the reactangle. Hint: $2 l+2 w=21$.
4. 

State the vertex and roots using any method. Round decimal roots to the hundredth.
3. $y=(x-9)(x+1)$
4. $y=(x-2)^{2}-4$

Vertex: $\qquad$

Roots: $\qquad$ \& $\qquad$ Roots: $\qquad$ \& $\qquad$
5. $y=x^{2}-5 x+1$
6. $y=4 x^{2}-9$

Vertex: $\qquad$ Vertex: $\qquad$

Roots: $\qquad$ \& $\qquad$ Roots: $\qquad$ \& $\qquad$
7. $y=-4.9 x^{2}+50.1 x-20.2$
8. $y=x^{2}-2 x+15$

Vertex: $\qquad$ Vertex: $\qquad$

Roots: $\qquad$ \& $\qquad$ Roots: $\qquad$ \& $\qquad$

Graph the equation below．Plot at least five points．

1．$y=x^{2}-12 x+29$


State the vertex and Roots for each equation below．Round decimal answers to the hun－ dredth．Write NO ROOTS is an equation has no real roots．Use any method you are com－ fortable with．

2－3．$y=(x-3)^{2}-16$
2．（Vertex）

3．（Roots） $\qquad$
4－5．$y=2 x^{2}-17 x-9$
4．（Vertex） $\qquad$

5．（Roots） $\qquad$
6－7．$y=-4.9 x^{2}+10 x+17$
6．（Vertex）

7．（Roots） $\qquad$

## Solve each. Round decimal answers to the hundredth.

8. The sum of two numbers is 21 and their product is 98 . Find the smaller number.
9. $\qquad$
10. The longer side of a rectangle is an inch longer than three times the length of the short side. If the area of the rectangle is 10 square inches, what is the length of the long side of the rectangle?

## 9.

A football is punted with an initial upward velocity of $\mathbf{7 2}$ feet per second when it left the kickers foot 4 feet above the ground.
10. Write an equation for this situation.
10.
11. How long will it take the ball to reach its maximum height?
11.
12. How long does it take the ball to land on the field?
12. $\qquad$
13. To the hundredth of a foot, how high does the ball get?
13. $\qquad$
14. To the hundredth of a second, how long does it take for the ball to reach a height of 50 feet on its way up?
14.

