## Understanding and interpreting graphs can be difficult. Uderstanding the relationship between the $x$ and $y$-axis is very important.

Example: The following graph shows the distance traveled by a school bus based on every morning from 6:30-7am.


1. What do the flat sections of the graph represent?
2. What do the steep sections of the graph represent?
3. Which is the best estimate of the buses average speed: $17 \mathrm{mph}, 20 \mathrm{mph}, 34 \mathrm{mph}$, 50mph?

Why doesn't path of the line ever go down?
Often you will need to recognize graphs plotting distance versus time, or distance from a point versus time, usually with time plotted on the x-axis.

Distance is called a Function of Time
Match each graph below with the appropriate situation:



a. Mary leaves her house, walks to the store, and returns with groceries.
b. Roger walks around the block.
c. Sylvia runs to the track, takes a few laps, and jogs home.

Use the two graphs below to answer the questions that follow:



1. When is Dan driving on the highway?
2. Approximate Dan's average speed for the trip to work.
3. Approximate Dan's top speed during the trip.
4. Who finishes the marathon first?
5. How do Megan and Carl's strategies compare?
6. Approximately how many minutes does it take Carl to run each mile during the first $1: 40$ of the race? ( $4,5,6$, or 7 minutes?)

These graphs look very different from graphs representing speed versus time.

Dan's Drive to Work


Dan's Drive to Work


## Use a sheet of graph paper to graph the following:

1. The distance you are from your locker from 7:20-12:20.
2. The total distance you have traveled during the same time period.
3. The speed you are moving during the same time period.

Be prepared to be able to explain your three graphs.

## Functions: Write this down and memorize it!

$y$ is a function of $x$ only if: for every domain value (x) there is exactly one range value ( y ).

This is easiest to see on a graph using the Vertical Line Test:
Examples: Which of the following are functions?




Examples: It is also easy to tell if a Relation (set of points) represents a function. Which is a function? Why or why not?

1. $(-5,4)(-3,2)(-1,1)(1,0)$
2. $(5,-2)(6,-3)(5,-4)(6,-5)$

Practice: Determine which relation represents a function:

1. $(-2,3)(-3,4)(-4,5)(-6,7)$
2. $(1,-3)(2,-3)(3,-3)(4,-3)$
3. $(-2,2)(2,-2)(2,2)(-2,-2)$
4. $(2,30)(4,50)(6,70)(8,90)$

Examples: It is more difficult to determine if an equation is a function for $y$ :
Explain why each equation belongs in the category it is in.
hint: Try plugging-in 4 for $x$ in each equation, see how many answers are possible for $y$.

Functions:

$$
\begin{array}{ll}
2 x-y=-3 & y=x^{2}-3 \\
y=|x-3| & x y=20
\end{array}
$$

Not Functions:

$$
\begin{array}{ll}
x=y^{2} & x=4 \\
x=|y| & y=\sqrt{x}
\end{array}
$$

## Review:

$y$ is a function of $x$ if:
Each $x$ gives one value for $y$.
The graph passes the vertical line test.

## Complete:

Every linear equation is a function unless its slope is $\qquad$ .
List an example of a linear equation that is NOT a function.

## Function Notation:

## If $y$ is a function of $x, y$ can be replaced with $f(x)$.

## Examples:

1. $y=2 x-3$ means exactly the same thing as $\qquad$ .
2. $y=x^{2}-3 x$ means exactly the same thing as $\qquad$ .

Do not get confused by function notation, if you get lost... switch $f(x)=$ back to $y=$.

In function notation, questions are often asked like this:
$f(x)=\frac{1}{2} x-3$ Find $f(4): \quad$ This is a fancy way of saying plug 4 in .
To denote different functions, you may often need to use $g(x)$ or $h(x)$.

## Practice:

Find the range of each function for the given domain:

1. $f(x)=5 x-7$
$\{D:-3,-2,-1,0\}$
2. $g(x)=x^{2}+2$
\{D: 9, 7, 5, 3\}

Challenge. Using both functions above, find $f(g(2))$ and $g(f(2))$.

For each pair below, answer whether each is a function. Write A. B. Both or Neither in the blank provided.
1.
A. $3 x-y=10$
B. $y=7$

1. $\qquad$
2. A. $(2,2)(3,3)(4,4)(5,5)$
B. $(2,1)(3,1)(4,1)(5,1)$
3. A. $y=|x|$
B. $x=|y|$
4. $\qquad$
5. $\qquad$
6. A. $x=y^{2}-y$
B. $y=x^{2}-x$
7. $\qquad$
8. 

A. $(0,1)(1,2)(2,3)(3,4)$
B. $(0,-1)(1,-2)(2,-3)(3,-4)$
5. $\qquad$
Use the functions below to answer the questions that follow:

$$
f(x)=x-5 \quad g(x)=2 x^{2} \quad h(x)=\frac{1}{2 x}
$$

6. $f(3)=$
7. $g(-2)=$
$\qquad$
8. $h(5)=$ $\qquad$
9. $h(x-5)=$ $\qquad$
10. $f(2)+g(2)=$ $\qquad$
11. $f(-4)=$ $\qquad$
12. $g(3)=$ $\qquad$

Find the range for the domain given:
15. $f(x)=2 x-3 \quad\{D: 7,5,-5,-7\}$
$\qquad$

Challenge. $f(x)=2 x^{2}-5 \quad\{D:-5<x<5\}$

## Tables and Functions:

You can create a table of values for a function just as you would any other equation.

## Review:

Create a table of values for the function $f(x)=5 x-7$

| $X$ | $f(x)$ |
| :--- | :--- |
|  |  |
|  |  |
|  |  |

You can go in reverse as well. Some functions are obvious.

## Practice:

Try to determine what function was used to create each table:

1. | x | $\mathrm{f}(\mathrm{x})$ |
| :--- | :--- |
| 2 | 12 |
| 3 | 13 |
| 4 | 14 |

$f(x)=$
2.

| $x$ | $f(x)$ |
| :--- | :--- |
| 5 | 10 |
| 7 | 14 |
| 9 | 18 |

$f(x)=$
$f(x)=$

Other times, it is not as easy.
For linear equations there is a method that will always work.

1. Find the slope. 2. Use Slope-Intercept Form and solve for b.

## Examples:

Try to determine what function was used to create each table:
1.

| $x$ | $f(x)$ |
| :--- | :--- |
| 2 | -9 |
| 5 | -15 |
| 6 | -17 |

$f(x)=$
$f(x)=$
$f(x)=$

Practice: Write a function for each table of values.
1.

| $x$ | $f(x)$ |
| :--- | :--- |
| -3 | -2 |
| 1 | -6 |
| 5 | -10 |

$f(x)=$
$f(x)=$

| $x$ | $f(x)$ |
| :--- | :--- |
| 6 | -1 |
| 3 | -3 |
| -9 | -11 |

## Some functions will not be linear.

If the slope of an equation is not constant, the function is not linear.
Look for squares, square roots, absolute value, and other common functions.
Practice: Write a function for each table of values.

1. | x | $\mathrm{f}(\mathrm{x})$ |
| :--- | :--- |
| 1 | -1 |
| 4 | -2 |
| 9 | -3 |

$f(x)=$

2. | $x$ | $f(x)$ |
| :---: | :--- |
| 3 | 3 |
| -4 | 4 |
| -6 | 6 |

$f(x)=$
3.

| $x$ | $f(x)$ |
| :--- | :--- |
| 1 | 1 |
| 2 | 0.5 |
| 4 | 0.25 |

$f(x)=$

Practice: Be careful, tables will not always look the same.
1.

| $f(x)$ | $x$ |
| :--- | :--- |
| -5 | 3 |
| -9 | 5 |
| 3 | -1 |

2. Three miles
in two hours.
Five miles in three hours.
Seven miles in four hours.
$f(x)=$
$f(h)=$
3. 

| $a$ | 4 | -4 | -8 |
| :--- | :--- | :--- | :--- |
| $f(a)$ | -4 | 2 | 5 |

$f(a)=$

Write a function for each table of values.
1.

| $x$ | $f(x)$ |
| :--- | :--- |
| 5 | 9 |
| 7 | 13 |
| 9 | 17 |

$f(x)=$
$f(x)=$

5. | $x$ | $f(x)$ |
| :--- | :--- |
| -9 | 0 |
| -6 | 2 |
| 3 | 8 |

$f(x)=$
8.

8. | x | $\mathrm{f}(\mathrm{x})$ |
| :--- | :--- |
| 4 | -1.8 |
| 3 | -2.1 |
| 2 | -2.4 |

$f(x)=$

7. | $x$ | $f(x)$ |
| :--- | :--- |
| 5 | -8 |
| -5 | -10 |
| -15 | -12 |
8. | $f(x)$ | $x$ |
| :--- | :--- |
| 3 | 3 |
| -5 | 7 |
| -13 | 11 |

$f(x)=$
$f(x)=$
$\mathrm{f}(\mathrm{x})=$
9.

| x | $\mathrm{f}(\mathrm{x})$ |
| :--- | :---: |
| 5 | 12.5 |
| 11 | 27.5 |
| -1 | -2.5 |

Write a function for each table of values. These are not linear.
10.

| $x$ | $f(x)$ |
| :--- | :--- |
| -2 | 5 |
| 5 | 26 |
| 12 | 145 |

$f(x)=$
$f(x)=$

| $x$ | $f(x)$ |
| :--- | :--- |
| -3 | 2 |
| -7 | 6 |
| 9 | 10 |

$f(x)=$

## Practice:

Try to determine what function was used to create each table:
1.

| $x$ | $f(x)$ |
| :--- | :--- |
| 5 | -9 |
| 3 | -5 |
| -1 | 3 |

$f(x)=$

2. | $x$ | $f(x)$ |
| :--- | :--- |
| 9 | -5 |
| 3 | -9 |
| -9 | -17 |

$f(x)=$
3.

| x | $\mathrm{f}(\mathrm{x})$ |
| :---: | :--- |
| 8 | 4 |
| 0 | 0 |
| -4 | -2 |

$f(x)=$

Functions can also be written to describe many real-world problems:

## Example:

Write a function that describes the cost to rent a widescreen television for the Superbowl if it costs $\$ 20$ plus $\$ 14.50$ a day. Cost is a function of $\qquad$ .

## Harder Example:

Write a function that describes the cost of gas if your car gets 30mpg and gas costs $\$ 3$ a gallon. Cost is a function of $\qquad$ .

## Practice:

1. Write a function that describes how far you can drive going 50 mph .

Distance is a function of $\qquad$ .
2. Write a function that describes the profit you make selling basketball tickets for $\$ 4$ each. Profit is a function of $\qquad$ .
3. Write a function that describes the profit you make selling basketball tickets for $\$ 6$ each if you already spent $\$ 100$ advertising the basketball game. Profit is a function of $\qquad$ .

## Practice:

1. Write a function that describes your math test score if you lose 6 points for every wrong answer (starting at 100). Score is a function of $\qquad$ .
2. Write a function that describes the change you will get back buying multiple $\$ 0.50$ candy bars with a $\$ 10$ bill. Change ( $\$$ ) is a function of $\qquad$ .
3. Write a function that describes the cost to have your vehicle towed if you are charged $\$ 15$ for the pickup plus $\$ 2$ for every mile.

Which of the following ARE functions (100 points each, -100 for every one you get wrong):
A. $(1,1)(2,2)(3,3)(4,4)$
B. $(1,2)(2,1)(3,4)(4,3)$
C. $(-1,4)(-2,4)(-3,4)(-4,4)$
D. $(-4,1)(-4,2)(-4,3)(-4,4)$
E. $(-2,4)(2,-4)(4,-2)(-2,4)$
F. $(5,10)(-5,10)(10,5)(-10,5)$

Which of the following ARE NOT functions (100 points each, -100 for every one you get wrong):
A.

B.

C.

D.


Which of the following ARE functions (200 points each, -200 for every one you get wrong):
A. $y=2 x-1$
B. $|y+1|=x$
C. $(y-1)^{2}=x$
D. $y=x^{2}$

Use the following functions to solve each:
$f(x)=-2 x-3$

$$
g(x)=x^{2}-1
$$

100. $f(3)=$
101. $g(-3)=$
102. $f(-3)+g(3)=$
103. $g(f(-2))=$

## Write a function for each:

100. 

| $x$ | $f(x)$ |
| :---: | :---: |
| 2 | -1 |
| 5 | 2 |
| 8 | 5 |

200. 

| $x$ | $f(x)$ |
| :--- | :--- |
| -3 | -2 |
| -6 | -6 |
| 9 | 14 |

300. 

| $x$ | $f(x)$ |
| :---: | :---: |
| 3 | -1 |
| 7 | 1 |
| -5 | -5 |

400. 

| $x$ | $f(x)$ |
| :---: | :---: |
| -8 | 9 |
| -2 | 3 |
| 1 | 2 |

Write a function for the following:
200. The score you get on a test if you lose 7 points for every wrong answer.
200. The cost of a canister of cashews if the charge is $\$ 5.50$ a pound plus $\$ 0.50$ for the canister.
200. The cost of a gym membership if there is a $\$ 50$ enrollment fee and the charge is $\$ 35$ a month.

For each pair below, answer whether each is a function. Write A. B. Both or Neither in the blank provided.
1.
A. $2 x-5 y=10$
B. $y=x-1$

1. $\qquad$
2. A. $(1,2)(0,3)(-1,2)(-2,3)$
B. $(2,1)(3,0)(2,-1)(3,-2)$
3. $\qquad$
4. 


4.
A. $x=y^{2}$
B. $y=x^{2}$
4. $\qquad$
5.
A. $(0,1)(1,2)(2,3)(3,4)$
B. $(0,-1)(1,-2)(2,-3)(3,-4)$
5. $\qquad$

Use the functions below to answer the questions that follow:

$$
f(x)=2 x-3 \quad g(x)=x(x-3) \quad h(x)=\frac{3}{x}
$$

6. $f(3)=$ $\qquad$
7. $g(-2)=$ $\qquad$
8. $h(0)=$ $\qquad$
9. $f(2)-g(2)=$
10. $g(5)+g(-5)=$ $\qquad$

Write a function for each table of values below:
11.

| $x$ | $f(x)$ |
| :--- | :---: |
| 5 | 1 |
| 3 | -3 |
| -1 | -11 |

12. 

| $x$ | $f(x)$ |
| :--- | :--- |
| 4 | 5 |
| 6 | 6 |
| 8 | 7 |

13. | $x$ | $f(x)$ |
| :--- | :--- |
| -3 | -3 |
| -6 | -1 |
| -9 | 1 |
14. $f(x)=$ $\qquad$
15. 

| $x$ | $f(x)$ |
| :--- | :---: |
| 5 | 0 |
| 11 | -12 |
| -2 | 14 |

15. 

| $x$ | $f(x)$ |
| :---: | :---: |
| 16 | 4 |
| 9 | 3 |
| 25 | 5 |

13
12. $f(x)=$ $\qquad$
13. $f(x)=$ $\qquad$
14. $f(x)=$ $\qquad$

Write a function for each and answer the questions that follow:
16. The distance traveled by a train traveling at 80mph.

Distance is a function of time in hours.
16. $\qquad$
17-18. The total amount spent at the fair if rides cost $\$ 1.50$ and the entrance fee is $\$ 7.50$.
17. Cost is a function of $\qquad$
18. Write the function $\qquad$

19-20. The profits from a raffle if tickets are sold for $\$ 5$ and $\$ 245.95$ was spent to set up the raffle.
19. Write the fiunction $\qquad$
20. How many tickets must be sold to guarantee you make at least $\$ 100$ profit? $\qquad$

